Coherent Thomson scattering

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Introduction

- ビームに密度分布を与え、対応する周波数のアンジュレータ、電磁波との衝突によりコヒーレント光を放出させる。
- HGHG
- Cooled HGHG
- EEHG
- Acceleration
- Thomson散乱による放出電磁波

- 35-100MeV電子でどこまで短波長光が得られるか。
- 発振は考えない

Beam-Laser interaction in an undulator

- Hamiltonian (for accelerator physics)

$$H = (1 + \delta) - \sqrt{(1 + \delta)^2 - \left(p_x - \frac{a_x}{\gamma_0}\right)^2 - \frac{1}{\gamma_0^2}}$$

$$\delta \equiv \frac{\Delta p}{p_0} \qquad z \equiv s - ct$$

$$a_x = \hat{a}_u \cos k_u s + \hat{a}_L \cos k_L z$$

$$\frac{dz}{ds} = -\frac{\left(p_x - \frac{a_x}{\gamma_0}\right)^2 + p_y^2 + \frac{1}{\gamma_0^2}}{p_s(p_s + 1 + \delta)} \qquad \frac{dx}{ds} = \left(p_x - \frac{a_x}{\gamma_0}\right)\frac{1}{p_s}$$

$$\frac{d\delta}{ds} = \frac{1}{\gamma_0 p_s}\left(p_x - \frac{a_x}{\gamma_0}\right)\frac{da_x}{dz} \qquad \frac{dp_x}{ds} = \frac{1}{\gamma_0 p_s}\left(p_x - \frac{a_x}{\gamma_0}\right)\frac{da_x}{dx}$$

- Solved by Runge-Kutta integration for example.

Symplectic expression (simplest)

- Expand H and take 2nd order (for $\frac{da_x}{dx} = 0$)

$$H = \left[-\frac{1+a_u^2}{2\gamma_0^2} + \frac{a_u}{\gamma_0}p_x\right]\delta + \left[\frac{1+a_u^2}{2\gamma_0^2}\right]\delta^2 + \frac{a_u a_L}{\gamma_0^2} - \frac{a_u}{\gamma_0}p_x + \frac{1}{2}\left(1 + \frac{1+3a_u^2}{2\gamma_0^2}\right)p_x^2 + \frac{p_y^2}{2}$$

$$\bar{\delta} = \delta - \frac{a_u}{\gamma_0^2}\frac{\partial a_L}{\partial z}\Delta s$$

$$\bar{z} = z - \left[\frac{1+a_u^2}{2\gamma_0^2} - \frac{1+a_u^2}{2\gamma_0^2}\bar{\delta} - \frac{a_u}{\gamma_0}\bar{p}_x\right]\Delta s$$

$$\bar{x} = x + \left[\frac{a_u}{\gamma_0}(1+\bar{\delta}) + \left(1 + \frac{1+3a_u^2}{2\gamma_0^2}\right)\bar{p}_x\right]\Delta s$$

$$\bar{p}_x = p_x$$

- Well-known 1D analytic equation is based on

$$\bar{\delta} = \delta - \frac{a_u}{\gamma_0^2}\frac{\partial a_L}{\partial z}\Delta s \qquad \bar{z} = z + \frac{1+a_u^2}{2\gamma_0^2}\bar{\delta}\Delta s \qquad z \equiv s - \beta_z ct$$

HGHG

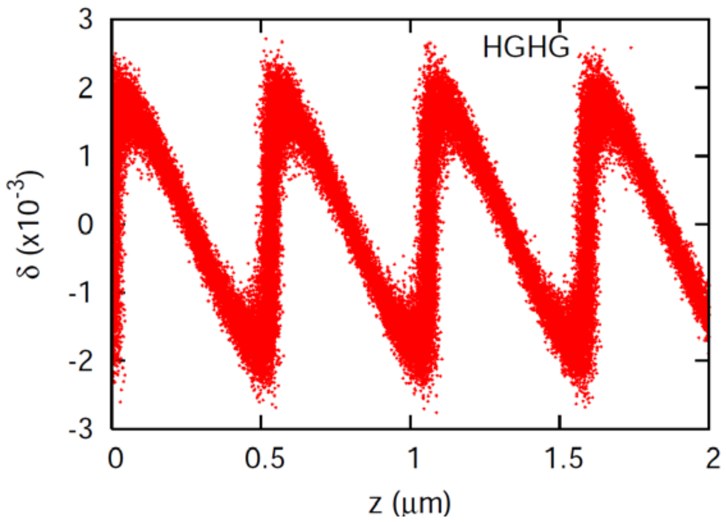
- Transformation

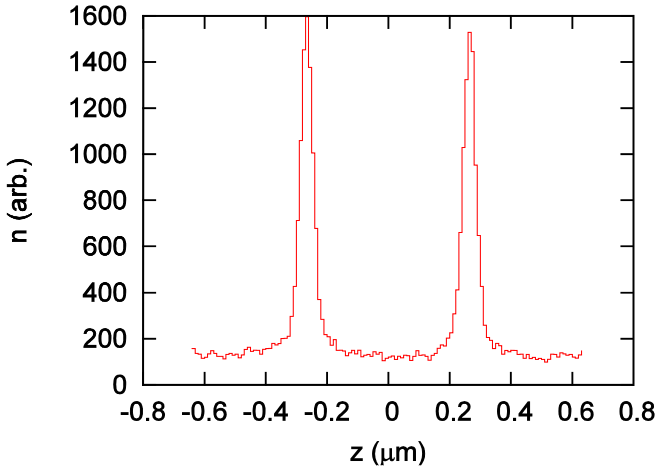
$$\delta \equiv \frac{\Delta p}{p_0} \qquad z \equiv s - ct \ \text{ or } z \equiv s - \beta_z ct$$

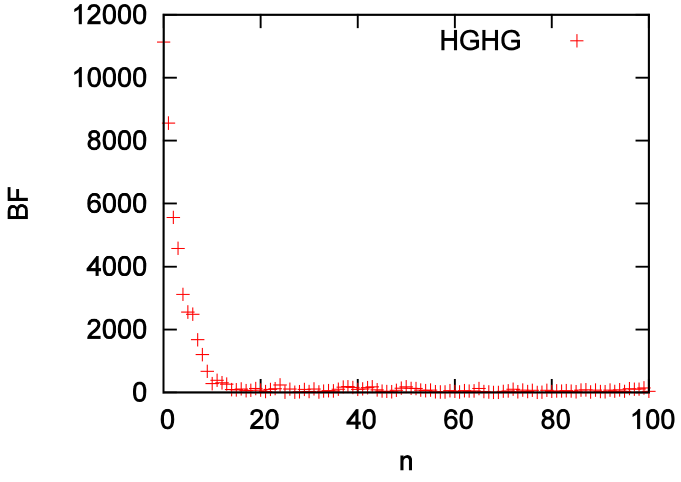
$$\delta_1 = \delta_0 + \Delta \sin k_L z_0 \qquad z_1 = z_0 + R_{56}\delta_1$$

- Bunching factor

$$|\rho_n| = \left| \int_0^{\lambda} \rho(z, s_1) e^{-ik_L n z} dz \right| = 2\pi \exp\left[-\frac{n^2 k_L^2 R_{56}^2 \sigma_\delta^2}{2}\right] J_n(nk_L R_{56}\Delta)$$







$$n \lesssim 10$$

Transformation in phase space for HGHG

- Continuity equation in phase space（Liouville theorem or Collisionless Vlasov equation）
- Map s_0 to s_1, $\boldsymbol{x}_1 = \boldsymbol{f}(\boldsymbol{x}_0)$, the distribution function is transferred as

$$\psi(\boldsymbol{x}, s_1) = \psi(\boldsymbol{f}^{-1}(\boldsymbol{x}), s_0)$$

For initial distribution $$\psi(\boldsymbol{x}, s_0) = \frac{1}{\sqrt{2\pi\sigma_\delta}} \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right)$$

- Map $\delta_1 = \delta_0 + \Delta \sin k_L z_0$ $\quad z_1 = z_0 + R_{56}\delta_1$
- Inverse map $\delta_0 = \delta_1 - \Delta \sin k_L (z_1 - R_{56}\delta_1)$ $\quad z_0 = z_1 - R_{56}\delta_1$
- Phase space distribution

$$\psi(\boldsymbol{x}, s_1) = \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp\left[-\frac{\{\delta - \Delta \sin k_L (z - R_{56}\delta)\}^2}{2\sigma_\delta^2}\right]$$

- Real space (z) distribution

$$\rho(z, s_1) = \int_{-\infty}^{\infty} \psi(\boldsymbol{x}, s_1) d\delta = \frac{1}{\sqrt{2\pi}\sigma_\delta} \int_{-\infty}^{\infty} \exp\left[-\frac{\{\delta - \Delta \sin k_L(z - R_{56}\delta)\}^2}{2\sigma_\delta^2}\right] d\delta$$

- Fourier components, bunching factor

$$\rho_n = \int_0^{\lambda_L} \rho(z, s_1) e^{-ik_L nz} dz = \frac{1}{\sqrt{2\pi}\sigma_\delta} \iint_{-\infty}^{\infty} \exp\left[-ik_L nz - \frac{\{\delta - \Delta \sin k_L(z - R_{56}\delta)\}^2}{2\sigma_\delta^2}\right] d\delta dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \int_{-\infty}^{\infty} \exp\left[-i(nx + nk_L R_{56}\sigma_\delta y) - \frac{(y - \Delta' \sin x)^2}{2}\right] dy\, dx$$

$\Delta' = \Delta/\sigma_\delta$

$$= \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{n^2 k_L^2 R_{56}^2 \sigma_\delta^2}{2}\right] \int_0^{2\pi} dx \exp[-inx - ink_L R_{56}\Delta \sin x] \int_{-\infty}^{\infty} \exp\left[-\frac{(y + ink_L R_{56}\sigma_\delta - \Delta' \sin x)^2}{2}\right] dy$$

$$= 2\pi \exp\left[-\frac{n^2 k_L^2 R_{56}^2 \sigma_\delta^2}{2}\right] i^n J_n(nk_L R_{56}\Delta)$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} dx e^{-inx + iy\sin x} = J_n(y)$$

$nk_L R_{56}\Delta \sim 1$

Cooled HGHG (symplectic system)

H. Deng and C. Feng, PRL111, 084801 (2013)

- Transverse Gradient Undulator is installed in a dispersive section.

$$a_x = a_u + a_L = \hat{a}_u\,(1 + \alpha x)\cos k_u s + \hat{a}_L \cos k_L z$$

$$\frac{da_x}{dx} = \hat{a}_u\, \alpha \cos k_u s$$

$$x = \eta\delta$$

$$a_u(\delta) = a_u(0)(1 + \alpha\eta\delta)$$

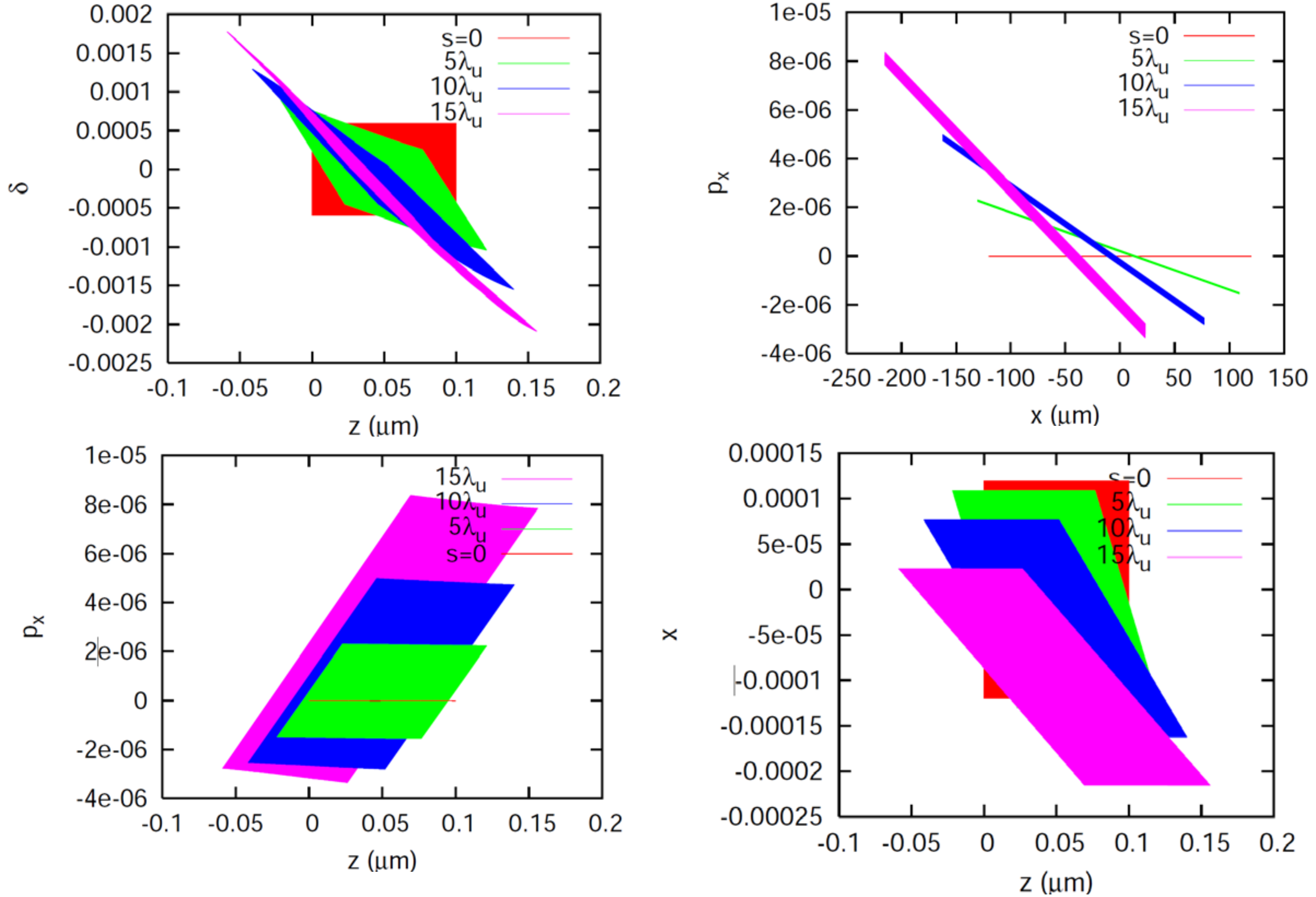
- Energy at the exit of the undulator

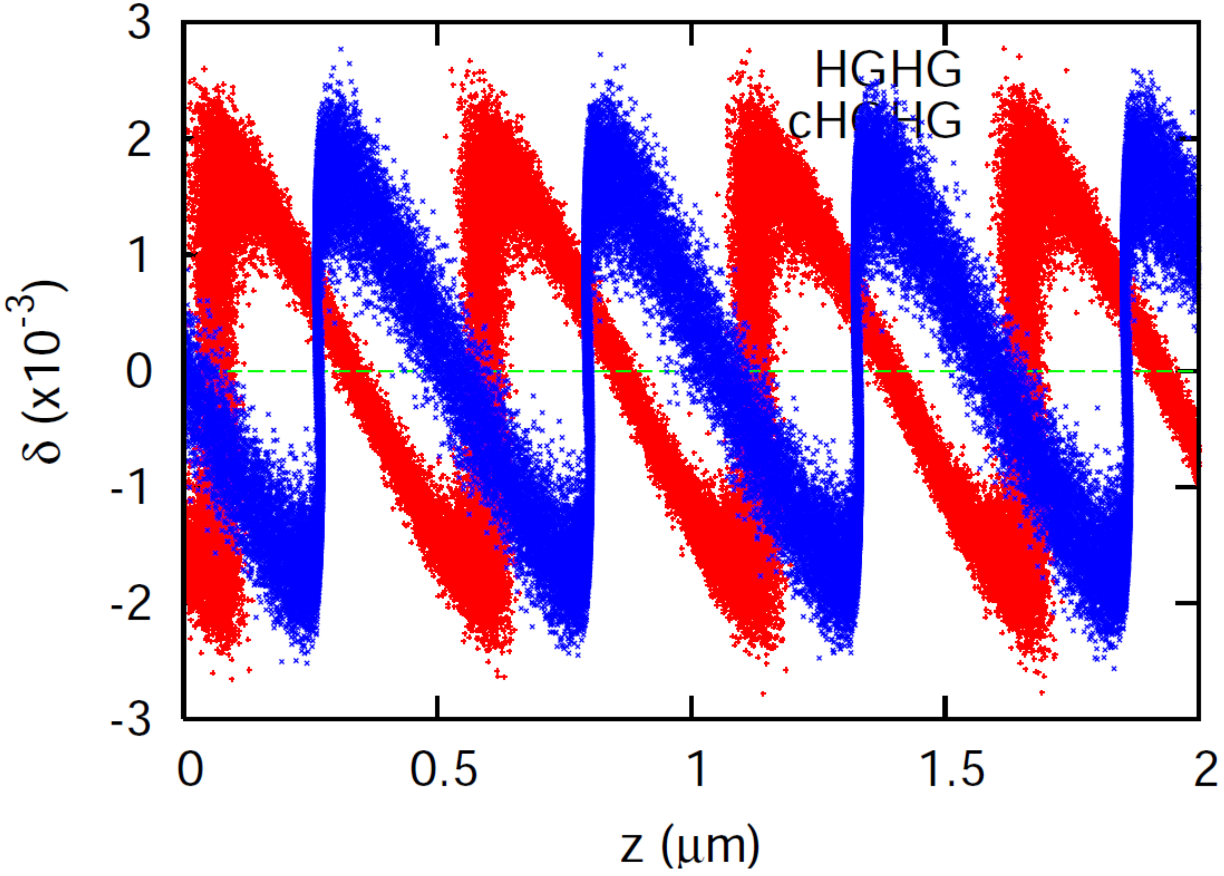
$$\alpha\eta = \frac{a_{u,0}^2 + 2}{a_{u,0}^2}\left(1 + \frac{1}{2\pi N_u \Delta}\right)$$

$$\rho_n = 2\pi \cancel{\exp\left[-\frac{n^2 k_L^2 R_{56}^2 \sigma_\delta^2}{2}\right]} i^n J_n(n k_L R_{56} \Delta)$$

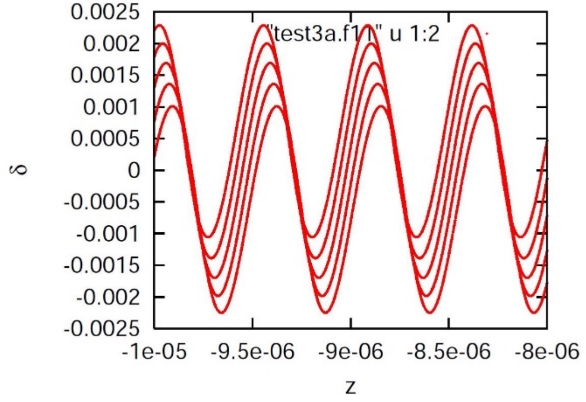
Fast drop term disappears

Evolution of phase space distribution





HGHG
cHGHG
δ (x10^-3)
z (μm)



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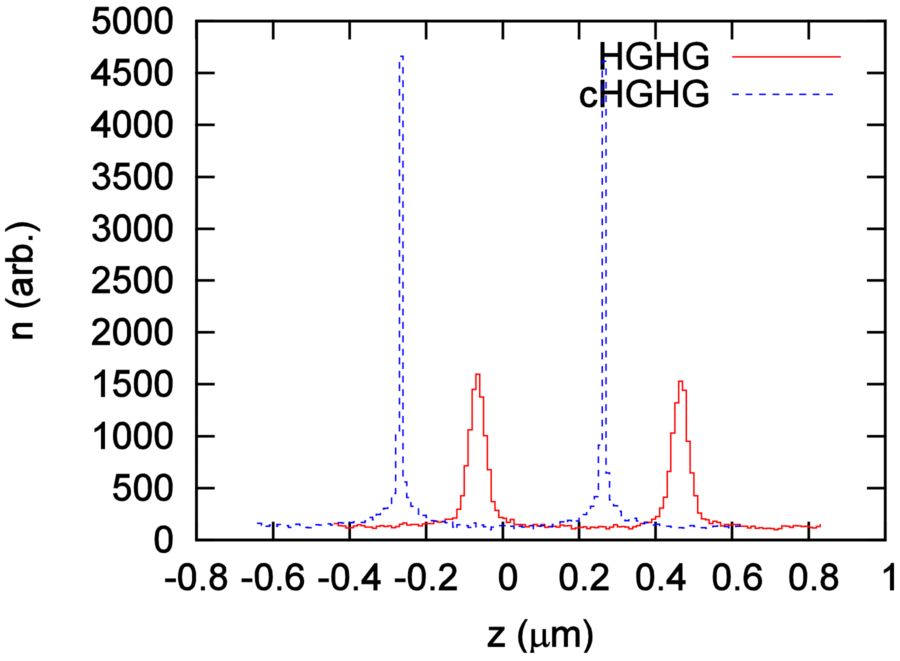
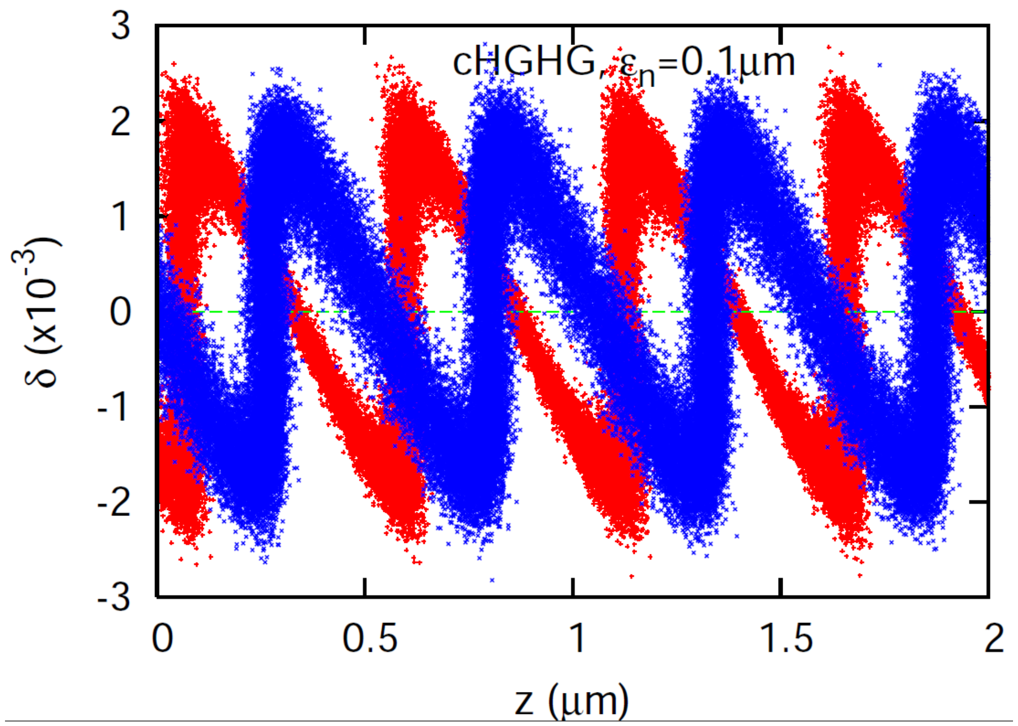
δ
z



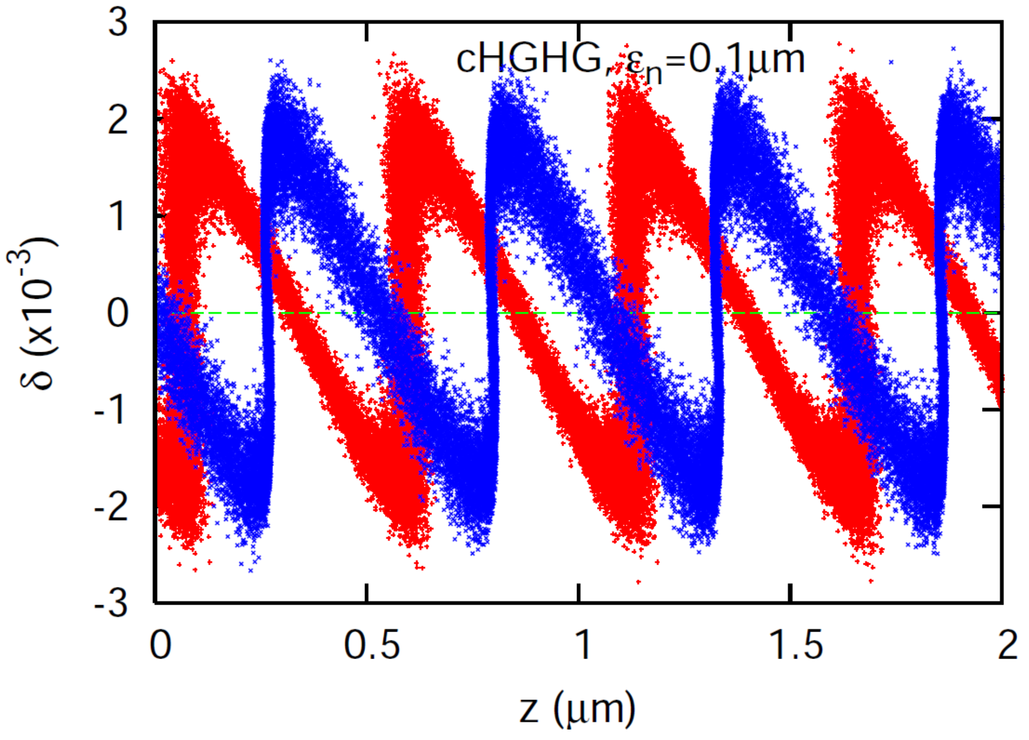
HGHG
cHGHG
n (arb.)
z (μm)

Effect of transverse emittance, $\varepsilon_n = 10^{-7}$ m

- Depend on dispersion, η.



η=0.2m α=79 m^{-1}

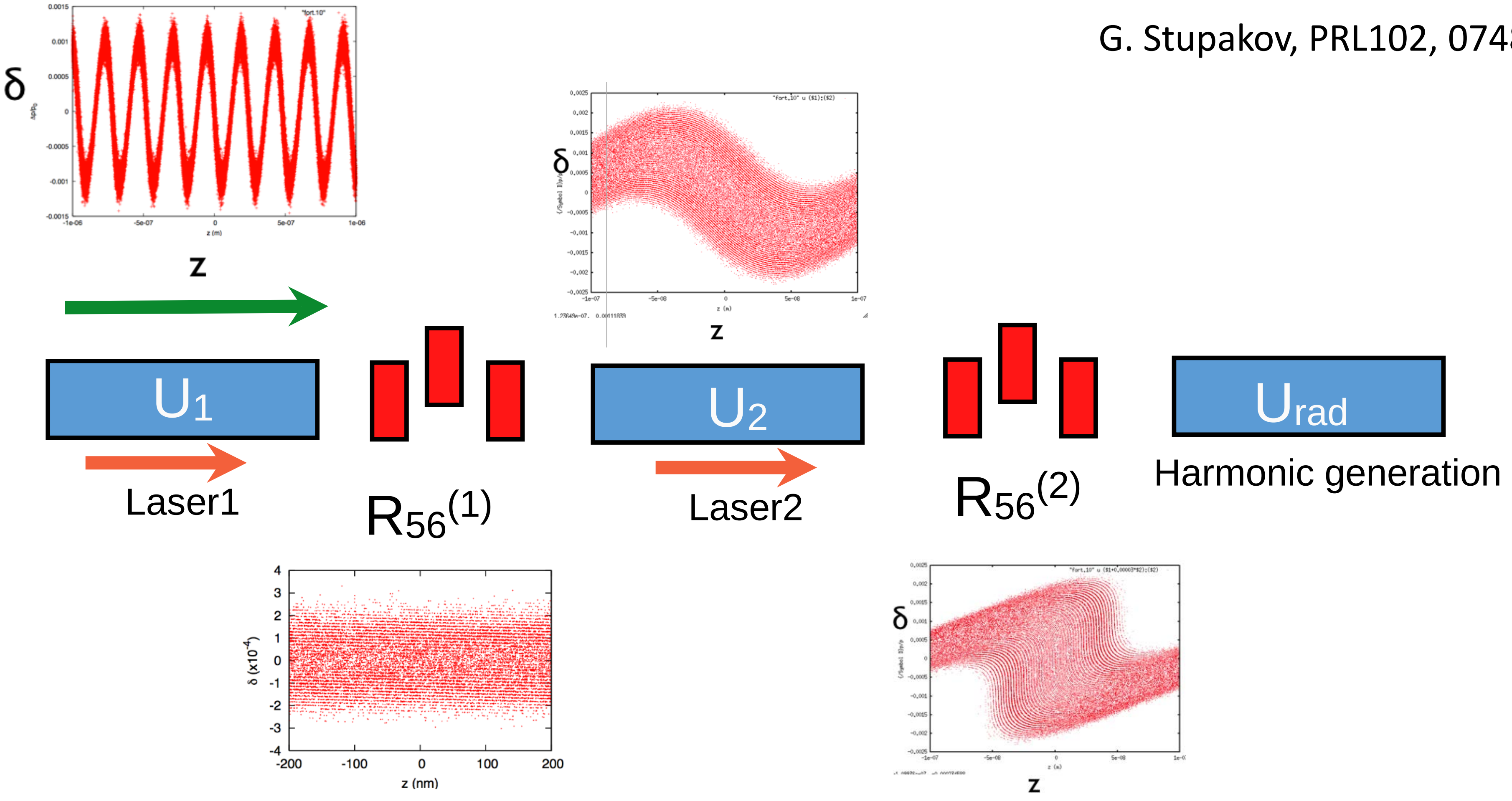


η=2m α=7.9 m^{-1}

EEHG

- アンジュレータとslippage section R_{56}の組み合わせ

G. Stupakov, PRL102, 074801 (2009)

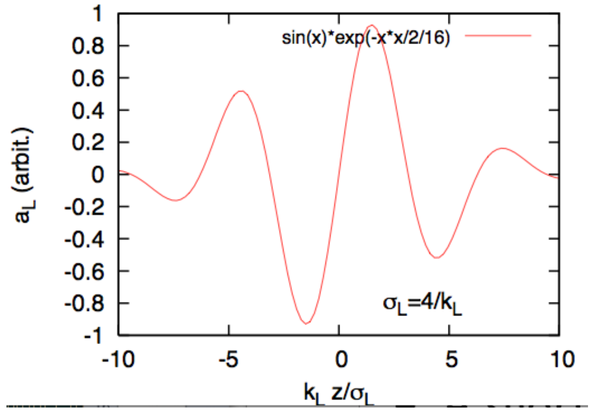


極短パルス発生

- 上述の方法でバンチ内に横縞(エネルギー方向)を作る。
- 短パルス(~10 fs)高強度レーザーを使い、ビームの一部に大きなChirpを作る。

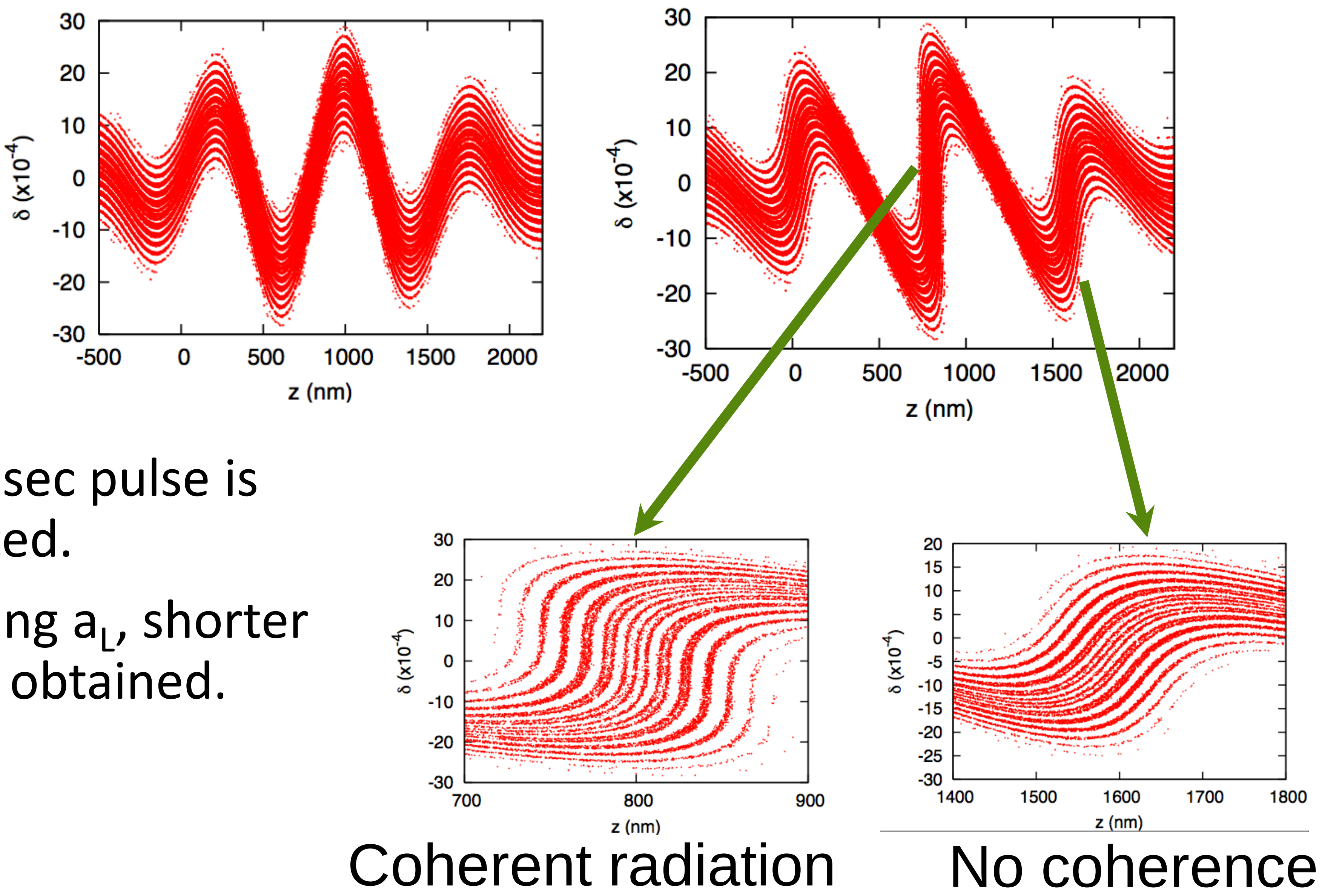
$$H = \frac{1}{2\gamma_z^2}\delta^2 + \frac{a_u a_L}{2\gamma^2} e^{-z^2/2\sigma_L^2} \cos(k_L z + \phi)$$

- Slippageにより縦縞に変換、attosecパルス発生



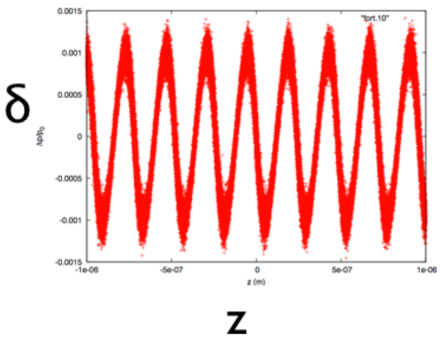
D. Xiang et al, PRSTAB12, 060701 (2009)

Simulation Example: 100 asec use EEHG

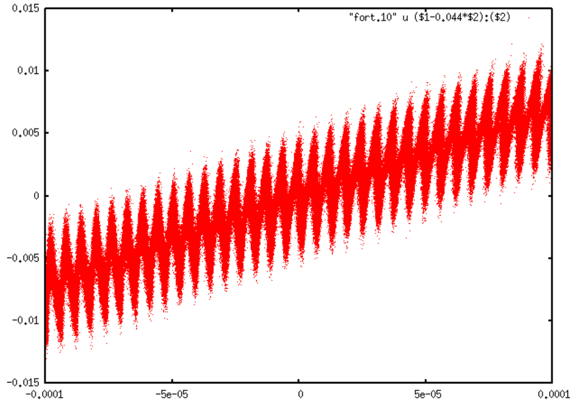


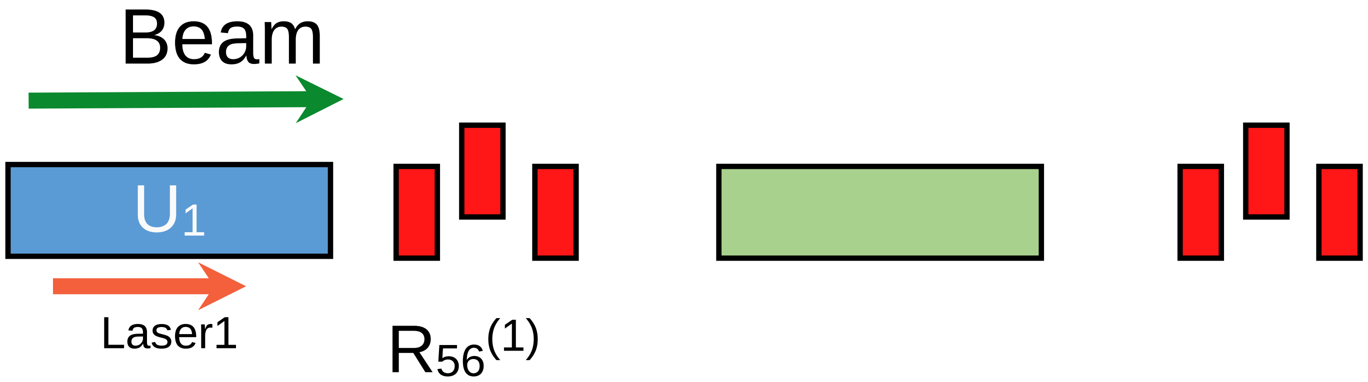
- Single asec pulse is generated.
- Increasing a_L, shorter pulse is obtained.

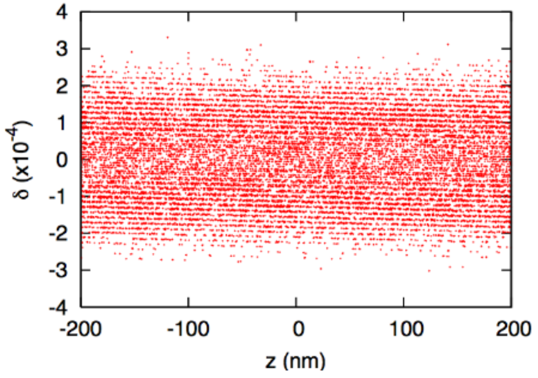
Acceleration

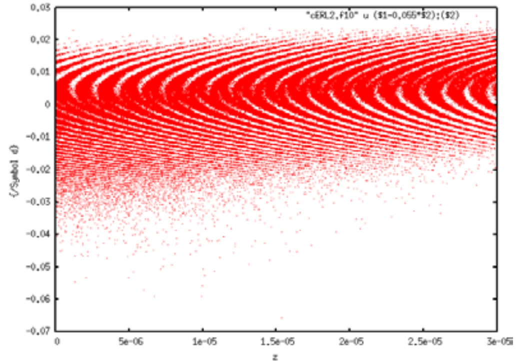


- 例
- 35MeV->100MeV
- 波長は25μm->5μm







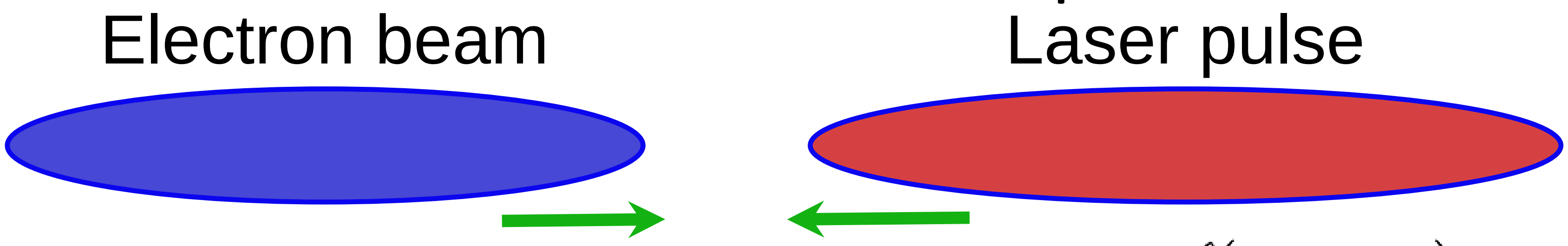


- 最初のSlippageを大きくすると短波長にできるが、非線形が出てくる

Coherent Thomson scattering

- Electron recoil energy << scattered photon energy in electron rest frame. Classical treatment is available, Thomson scattering.
- Electrons, which oscillate in the electro-magnetic field of (laser) pulse, emit radiation.
- For Pre-micro-bunched beam, coherent Thomson scattering

Collision of two beam pulse

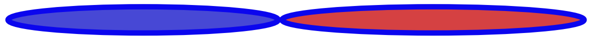


$$s_{min,max} = \mp \frac{\ell(\sigma_z + \sigma_L)}{2}$$

- ct_{min} :

$$z_i = \ell\sigma_z \qquad s = \frac{\ell(\sigma_z - \sigma_L)}{2}$$

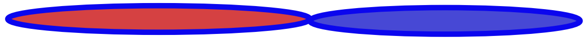
$$ct = s - z_i + \sqrt{x_o^2 + y_o^2 + (s_o - s)^2}$$



$$ct_{min} = s_o - \ell\sigma_z$$

- ct_{max} :

$$z_i = -\ell\sigma_z \qquad s = -\frac{\ell(\sigma_z - \sigma_L)}{2}$$



$$ct_{max} = \frac{\ell(\sigma_z + \sigma_L)}{2} + \sqrt{x_o^2 + y_o^2 + \left\{s_o + \frac{\ell(\sigma_z - \sigma_L)}{2}\right\}^2}$$

Hamiltonian for particle-laser interaction (classical)

- Colliding photon, traveling -s direction

$$H = (1+\delta) - \sqrt{(1+\delta)^2 - (\boldsymbol{p} - \frac{\boldsymbol{a}}{\gamma})^2 - \frac{1}{\gamma^2}} + \frac{a_z}{\gamma}$$

$$\begin{aligned} a_x &= a_0 \exp\left[-\frac{(s+ct)^2}{2\sigma_L^2} - ik(s+ct)\right] \\ &= a_0 \exp\left[-\frac{(2s-z)^2}{2\sigma_L^2} - ik(2s-z)\right] \end{aligned}$$

$$a_y = 0$$

Equation of motion for particles

$$x' = \frac{\partial H}{\partial p_x} = \frac{p_x - \frac{a_x}{\gamma}}{p_s} \qquad y' = \frac{\partial H}{\partial p_y} = \frac{p_y - \frac{a_y}{\gamma}}{p_s}$$

$$p_x' = -\frac{\partial H}{\partial x} = 0 \qquad p_y' = -\frac{\partial H}{\partial y} = 0$$

$$z' = \frac{\partial H}{\partial \delta} = 1 - \frac{1+\delta}{p_s}$$

$$p_s \equiv \sqrt{(1+\delta)^2 - \left(\boldsymbol{p} - \frac{\boldsymbol{a}}{\gamma}\right)^2 - \frac{1}{\gamma^2}}$$

$$\delta' = -\frac{\partial H}{\partial z} = \frac{1}{\gamma p_s}\left(p_x - \frac{a_x}{\gamma}\right)\frac{\partial a_x}{\partial z}$$

- The trajectory is solved by Runge-Kutta method.
- The trajectory has to be represented by function of t to be used in Feynman expression.

$$t = t_i + \frac{R}{c} \Rightarrow s - z_i + R - ct = 0$$

Radiation

$$\boldsymbol{E}_i(\boldsymbol{x}_o,t) = \frac{e}{4\pi\varepsilon_0}\left[\frac{\boldsymbol{R}_i}{R_i^3} + \frac{R_i}{c}\frac{d}{dt}\left(\frac{\boldsymbol{R}_i}{R_i^3}\right) + \frac{1}{c^2}\frac{d^2}{dt^2}\left(\frac{\boldsymbol{R}_i}{R_i}\right)\right]$$

$$\boldsymbol{R}_i = \underset{\text{observer}}{\boldsymbol{x}_o} - \underset{\text{electrons}}{\boldsymbol{x}_i} = \begin{pmatrix} x_o - x_i \\ y_o - y_i \\ s_o - s_i \end{pmatrix} \qquad \boldsymbol{n} = \frac{\boldsymbol{R}_i}{R_i}$$

$$\boldsymbol{E}(\boldsymbol{x}_o,t) = \sum_{i=0}^{N-1} \boldsymbol{E}_i(\boldsymbol{x}_o,t)$$

$$t = t_i + \frac{R_i}{c} \qquad z_i = s - ct_i$$

- t: Observer time, $R_i(t)$: motion seen by the observer
- Determine t_i and s for given t and z_i,.

s(t), z_i(t) for each particle

- New Raphson method

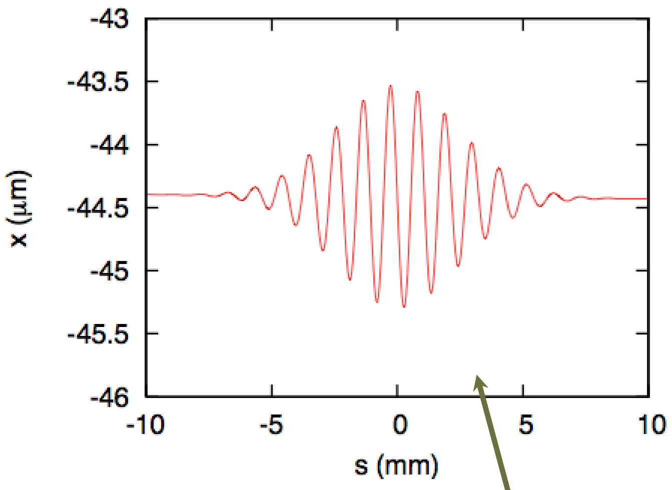
Assume $x_i \ll x_o$ $y_i \ll y_o$

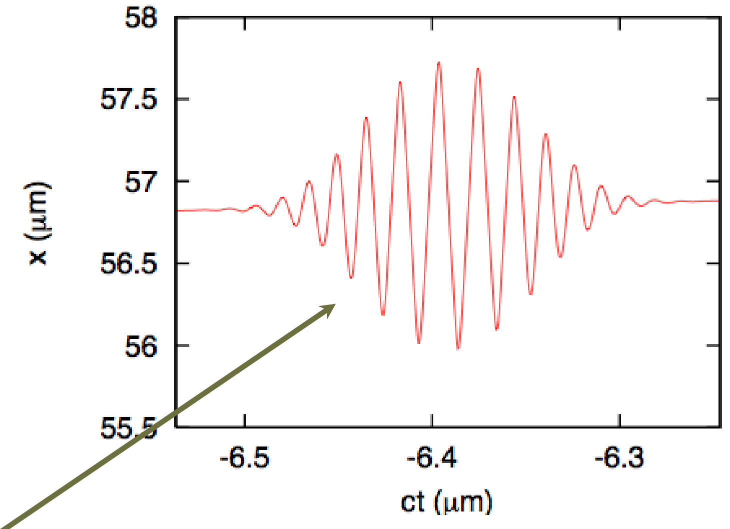
$$f(s, z_i(s), ct) = s - z_i(s) + \sqrt{(x_o - x_i(s))^2 + y_o^2 + (s_o - s)^2} - ct$$

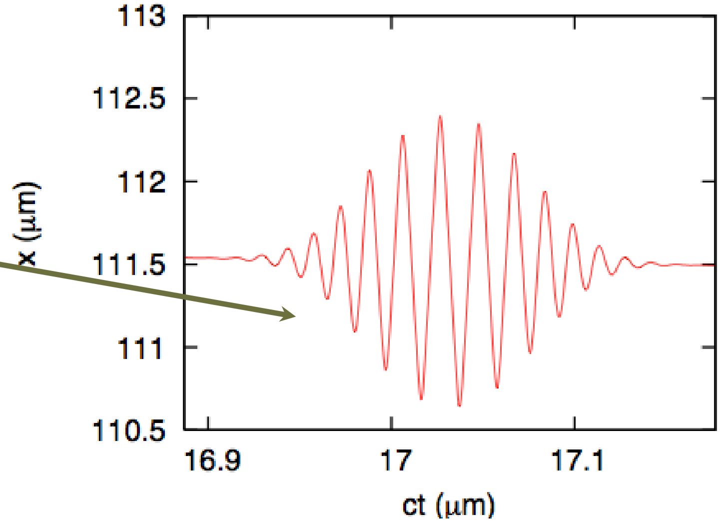
$$f_i(s, z_i(s), ct) = s - z_i(s) + \sqrt{x_o^2 + y_o^2 + (s_o - s)^2} - ct = 0$$

$$f_i'(s, z_i(s), ct) = 1 - z_i'(s) - \frac{s_o - s}{\sqrt{x_o^2 + y_o^2 + (s_o - s)^2}}$$

Beam orbit in Thomson scattering

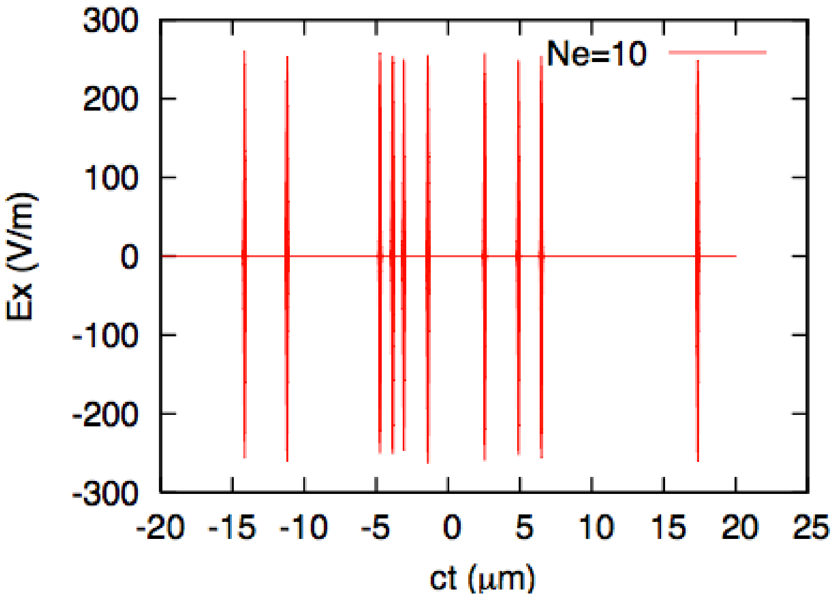






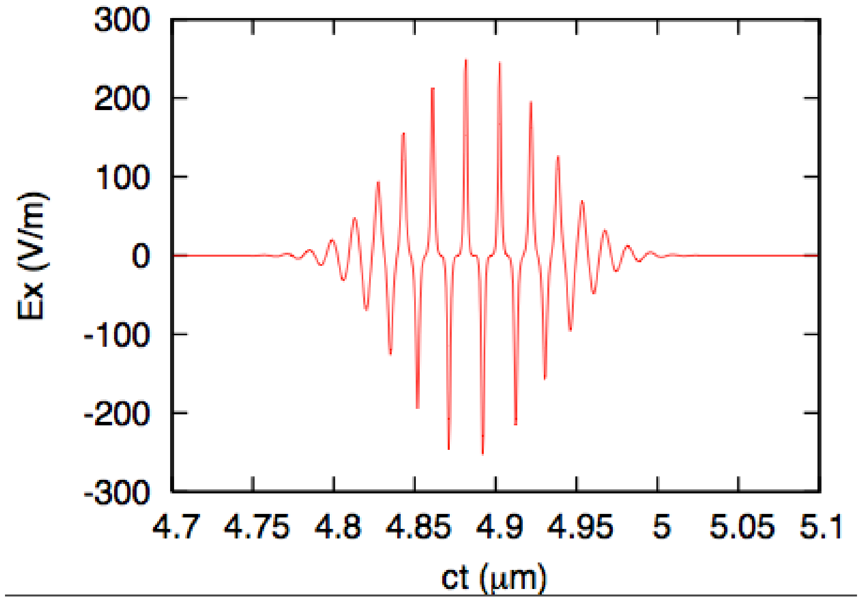
- Beam motion along s.
- Beam motion in the observer time.

Electric field produced by sampled electrons

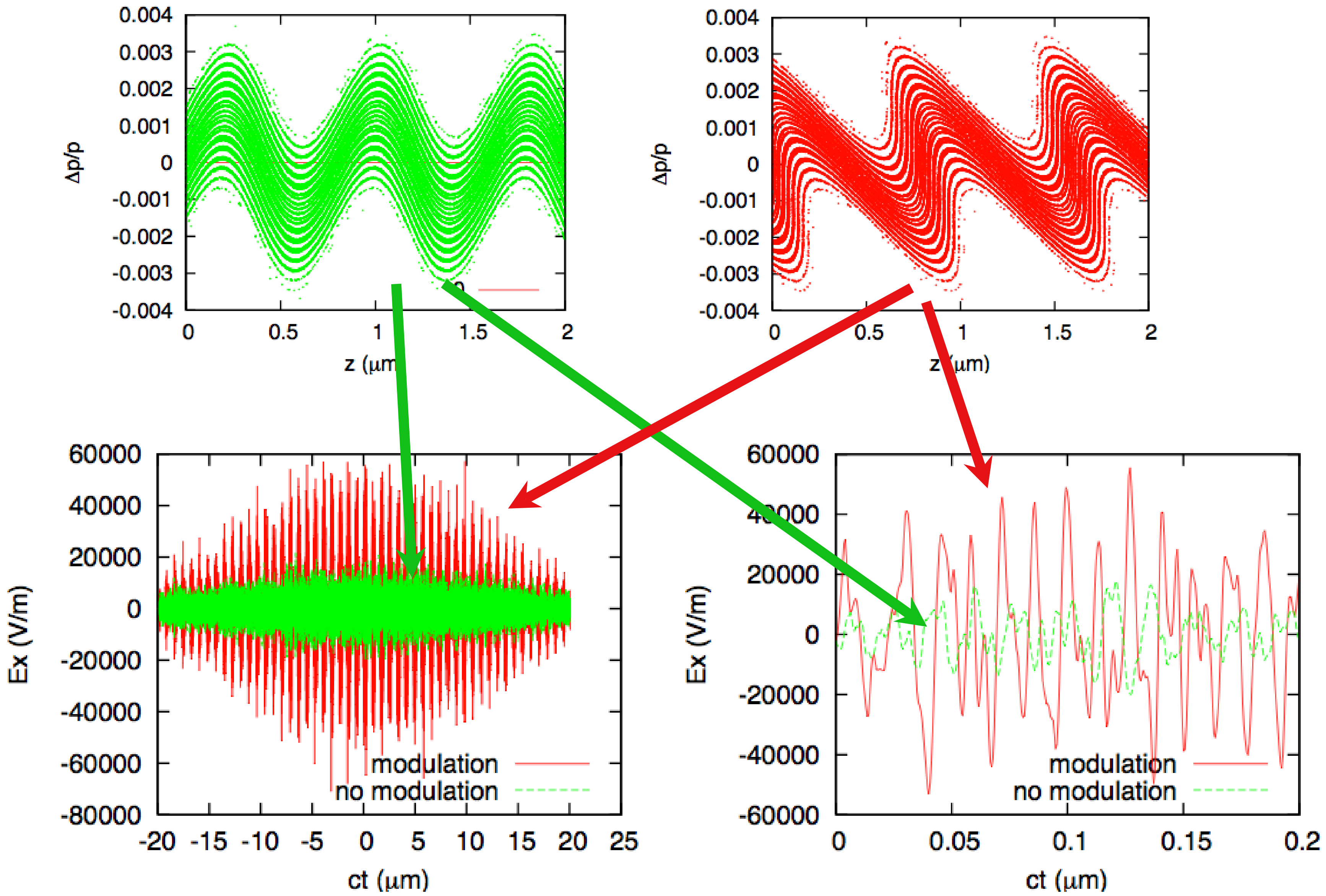


10 electrons

Detailed electric field by 1 electron



Electric field for the echo beam



Atto pulse generation

- Modulator $\Delta z/c$=50 asec
  - Laser λ_L=800 nm, a_L= 1.3x10^{-3}.
  - Undulator λ_u=2cm, a_u=1.46, L_u=4 cm, B=0.77T
  - Slippage
- Colliding laser λ_L=0.6mm

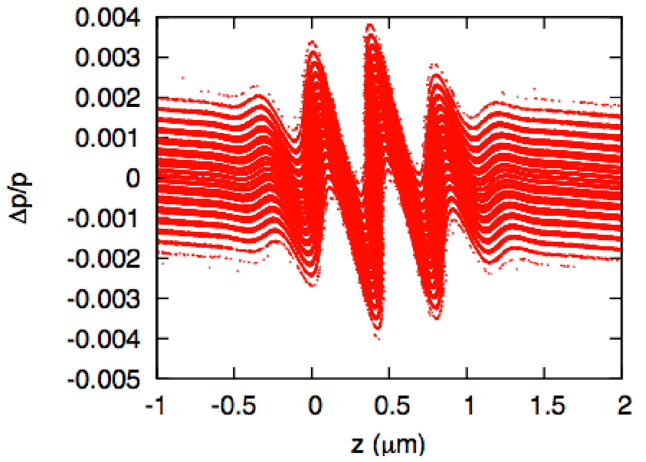
$$\Delta z = \frac{2\gamma^2}{k_L^2 a_u a_L L_u}\Delta\delta$$

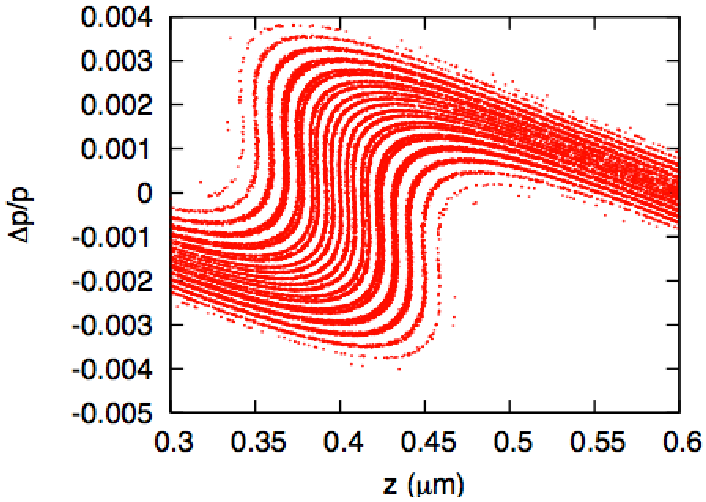
$\Delta\delta$ before 3rd modulation

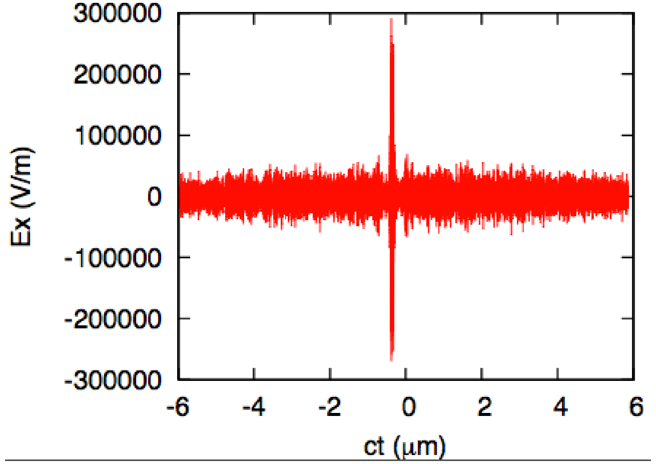
$$\Delta\delta = 3\sigma_\delta$$

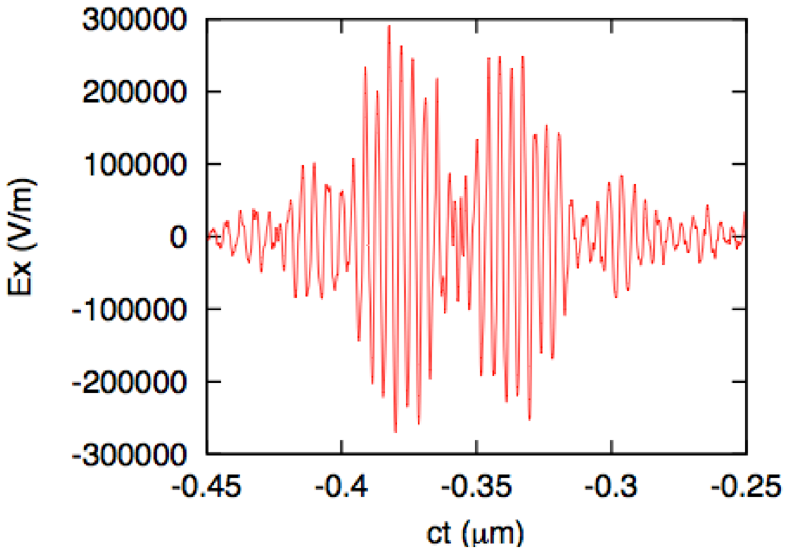
$$R_{56}^{(3)} = \left(\frac{d\delta}{dz}\right)^{-1} \qquad \frac{d\delta}{dz} = \frac{k_L^2 a_u a_L}{2\gamma^2} L_u$$

Atto pulse generation









Summary

- Pre-micro-bunching using HGHG, cooled-HGHG, EEHG is evaluated.
- Coherent Thomson scattering with the pre-micro-bunched beam is evaluated.
- Very short pulse is generated by Thomson scattering.