CSR calculation using mesh method

Demin Zhou

Thanks to: N. Nakamura, K. Ohmi, T. Agoh, G. Stupakov, K. Yokoya, M. Shimada, K. Oide, M. Kikuchi, H. Ikeda, N. Iida, ...

第63回ERLビームダイナミックスＷＧミーティング, KEK

Nov. 09, 2011

Outline

1. Introduction to CSRZ code

2. Field dynamics of CSR

3. Application to SuperKEKB DR

4. Application to cERL@KEK

5. Summary

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Field equations

Parabolic equation in Frenet-Serret coordinate system: $a/R \ll 1$

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp - \frac{1}{\epsilon_0}\nabla_\perp \rho_0 + 2k^2\left(\frac{x}{\underline{R(s)}} - \frac{1}{\underline{2\gamma^2}}\right)\vec{E}_\perp\right]$$

Longitudinal field:

$$E_s = \frac{i}{k}\left(\nabla_\perp \cdot \vec{E}_\perp - \mu_0 c J_s\right) \qquad J_s = \rho_0 c$$

Longitudinal impedance:

$$k \equiv \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$Z(k) = -\frac{1}{q}\int_0^\infty E_s(x_c, y_c)\,ds$$

Field separation:

$$\vec{E}_\perp = \vec{E}_\perp^r + \vec{E}_\perp^b \longrightarrow \frac{\partial \vec{E}_\perp^r}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp^r + 2k^2\left(\frac{x}{R(s)} - \frac{1}{2\gamma^2}\right)\left(\vec{E}_\perp^r + \vec{E}_\perp^b\right)\right]$$

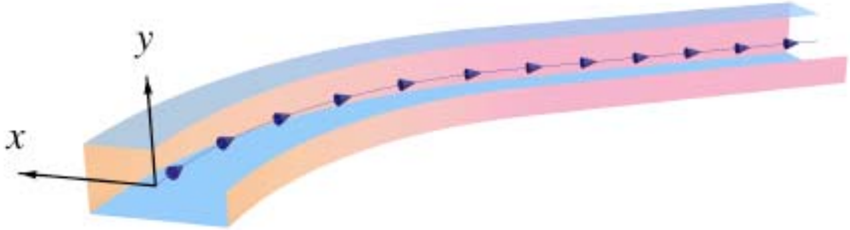
T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004).

Model for numerical calculation

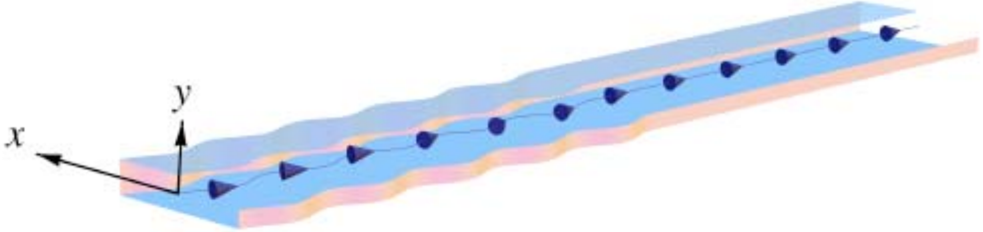
1. The curvature is variable (a series of dipoles, wiggler, etc.)
2. Chamber cross-section along the beam orbit:

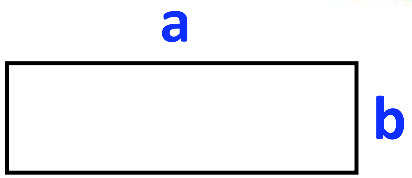
Uniform rectangular cross-section (2D)

Single dipole



Wiggler - “Wiggling pipe”





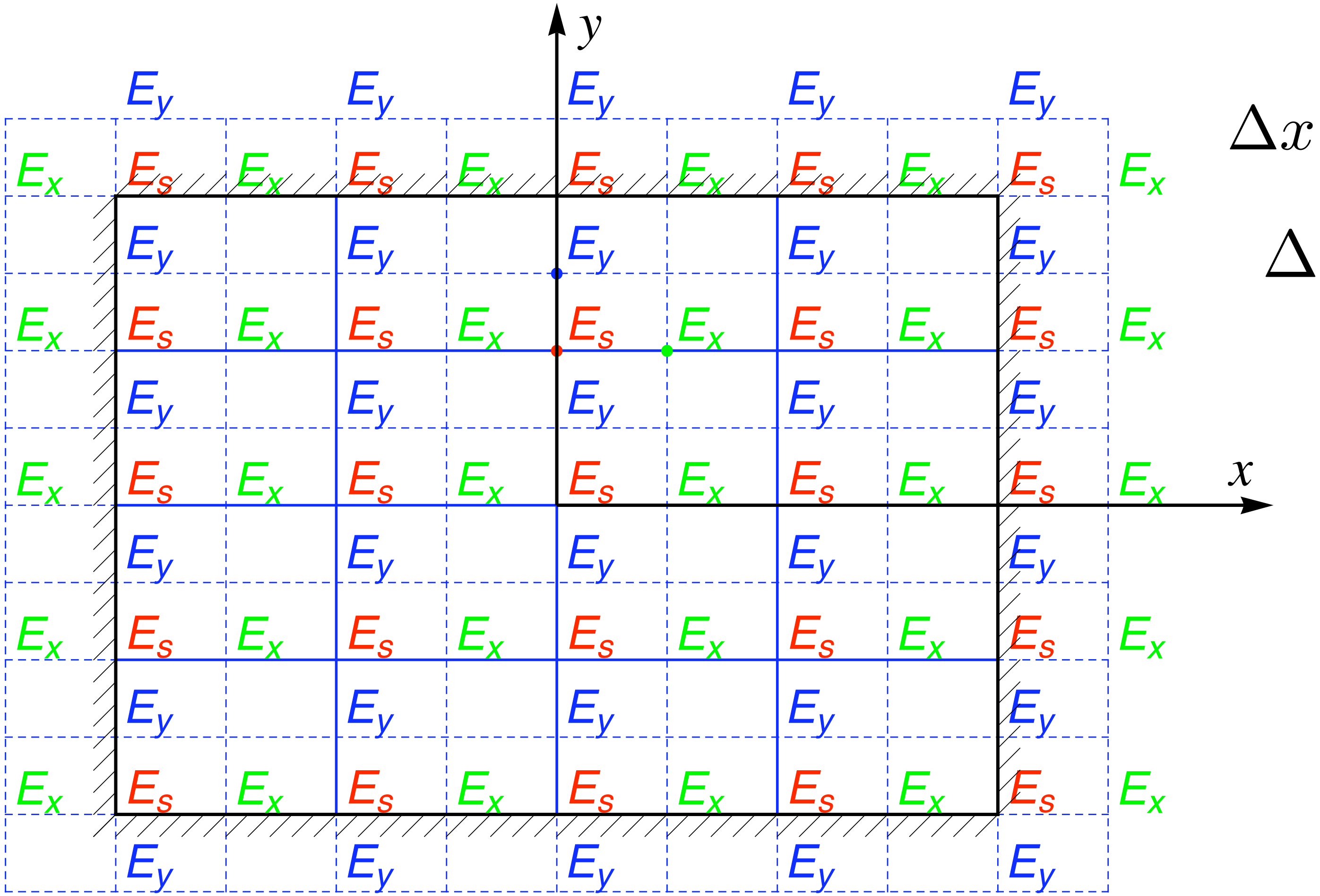
G. Stupakov and D. Zhou, SLAC-PUB-14332
D. Zhou, PhD thesis, KEK, 2011

Beam line: Line=(Bend)
=(Bend1, Bend2, ...)
=(Bend, Drift)
=(Bend1, Drift1, Bend2, Drift2, ...)

Numerical scheme

Finite-difference discretization:

1. Staggered grid: Central difference → Avoid numerical oscillations
2. Ghost points: Boundary conditions → Avoid numerical damping



$$\Delta x, \Delta y \ll (R/k^2)^{1/3}$$

$$\Delta s \ll (R^2/k)^{1/3}$$

T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004).
D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

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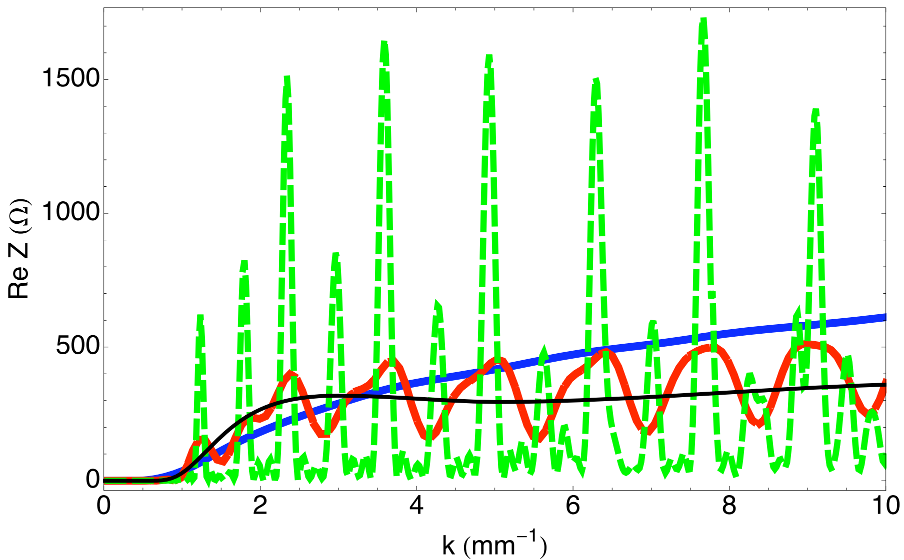
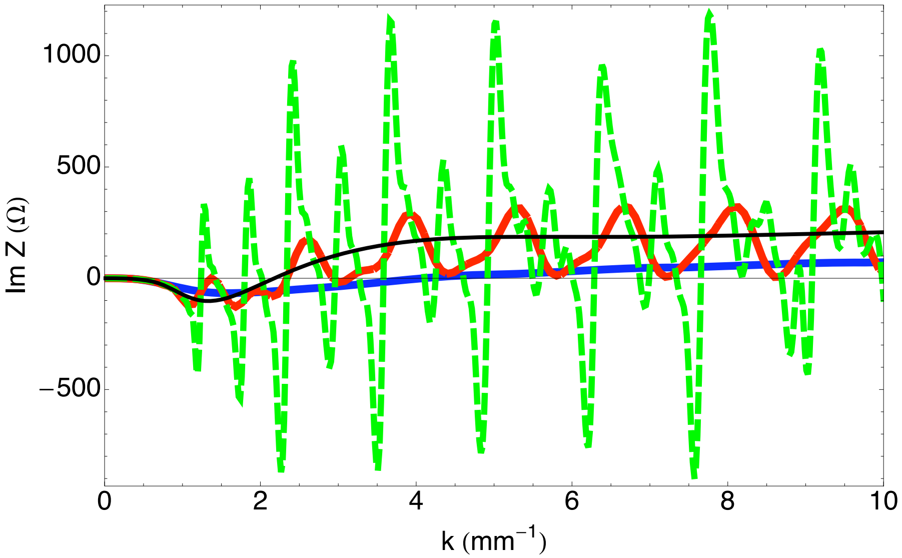
Excited modes in a long toroidal pipe

Single dipole:

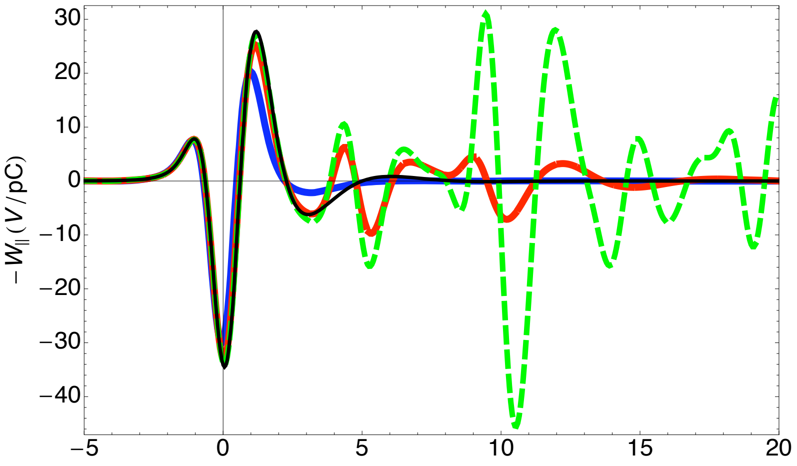
a/b=60/30 mm, R=5 m, L_{bend}=0.5/2/8 m   Bending angle=0.1/0.4/1.6 rad



Real part



Imaginary part



Wake potential

$$\sigma_z = 0.5 \text{ mm}$$

Blue solid lines: L_{bend}=0.5 m

Red dashed lines: L_{bend}=2 m

Green dotted lines: L_{bend}=8 m

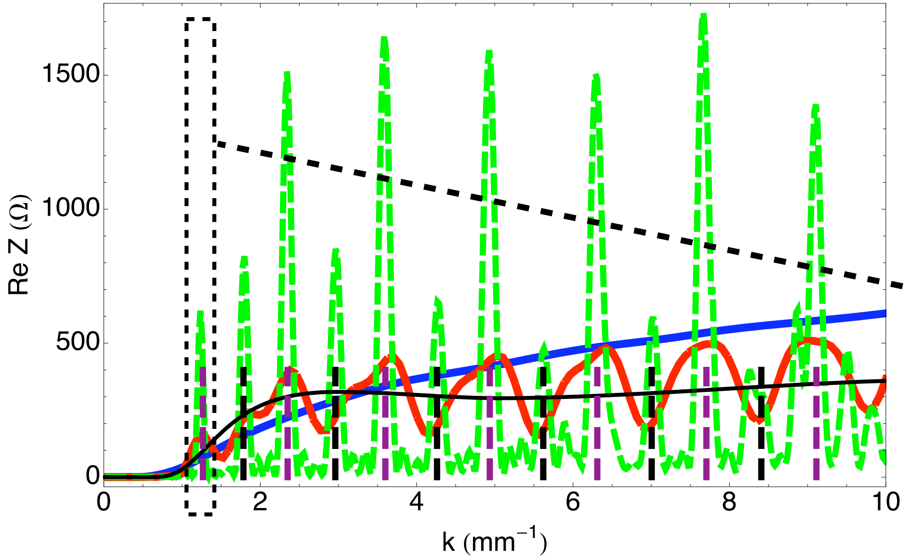
Black solid lines: Parallel plates model

D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

Related to eigenmodes

Radiation fields (E_x^r) on resonance poles (p=1)

a/b=60/30 mm, L_{bend}=8 m, R=5 m

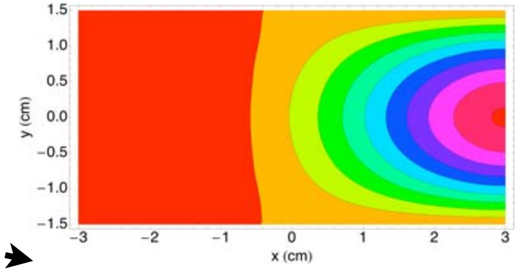


Freq. & index

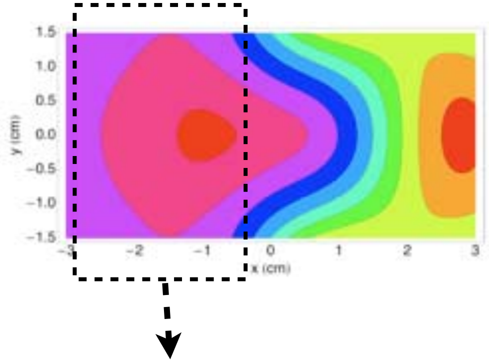
$$k = 1230\ \mathrm{m}^{-1}$$

$$(m,p) = (0,1)$$

Im. E_x^r:

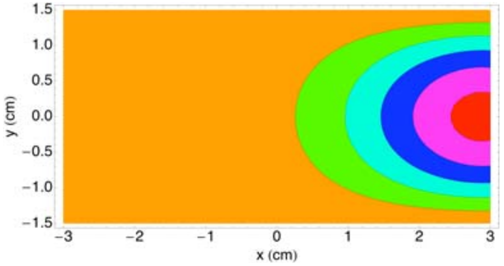


Re. E_x^r:



Beam self-field

Mode pattern:



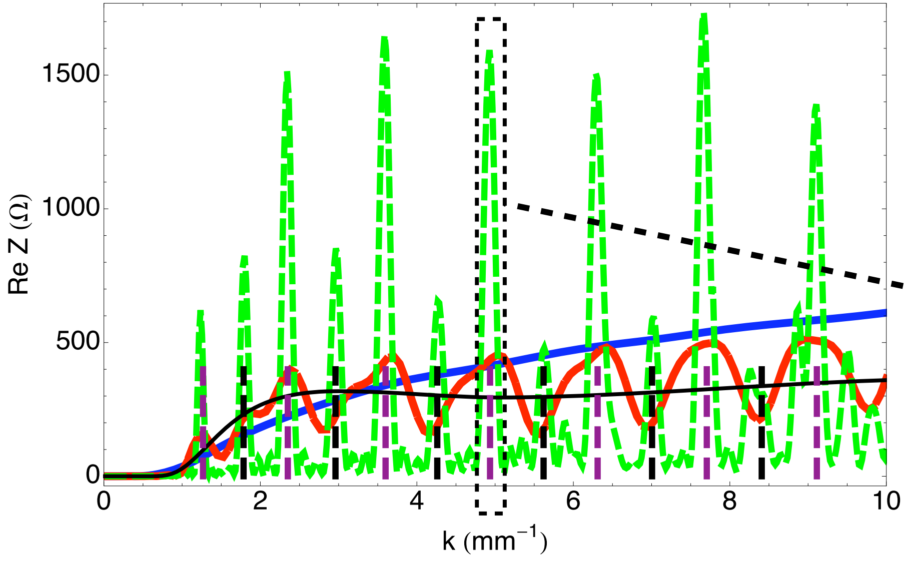
E_x mode: $$E_x(x,y) = E_{x0}\mathrm{Ai}\left(k_y^2\kappa^2 - x/\kappa\right)\sin\left[k_y(y+b/2)\right]$$

G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

Related to eigenmodes (cont’d)

Radiation fields (E_x^r) on resonance poles (p=1)

a/b=60/30 mm, L_{bend}=8 m, R=5 m

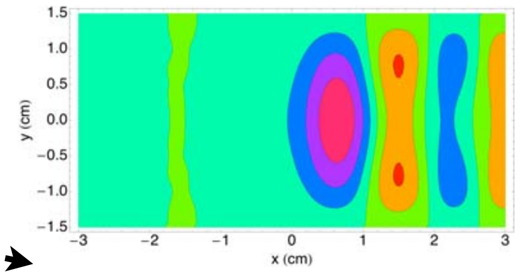


Freq. & index

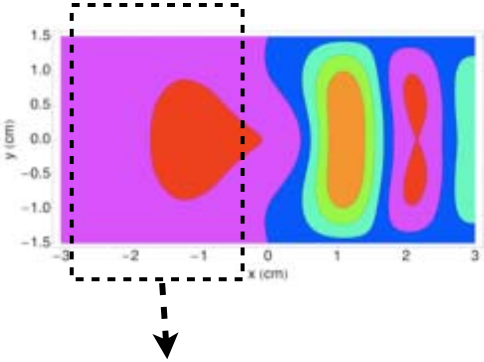
$$k = 4930\ \mathrm{m}^{-1}$$

$$(m,p) = (3,1)$$

Im. E_x^r:

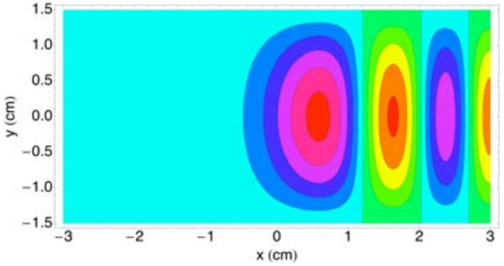


Re. E_x^r:



Beam self-field

Mode pattern:



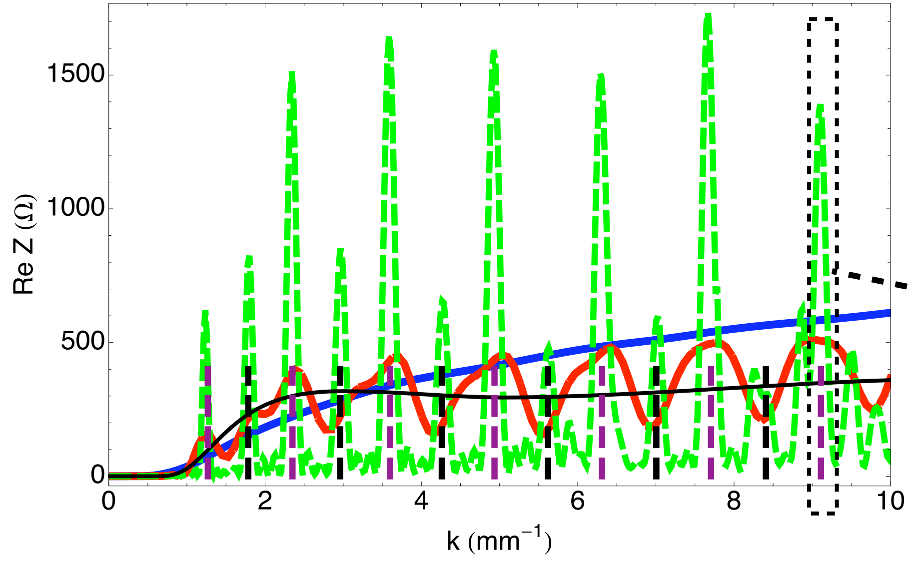
E_x mode: $E_x(x,y) = E_{x0}\mathrm{Ai}\left(k_y^2\kappa^2 - x/\kappa\right)\sin\left[k_y(y+b/2)\right]$

G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

Related to eigenmodes (cont'd)

Radiation fields (E_x^r) on resonance poles (p=1)

a/b=60/30 mm, L_{bend}=8 m, R=5 m

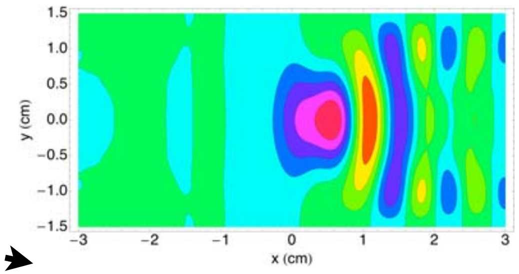


Freq. & index

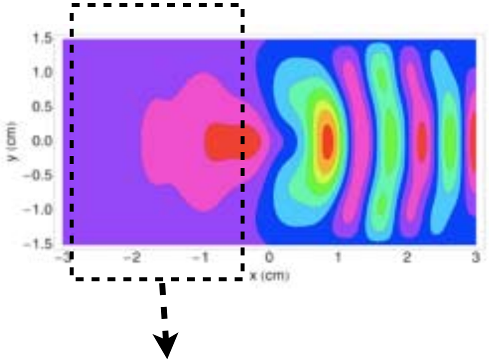
$$k = 9100\ \mathrm{m}^{-1}$$

$$(m, p) = (6, 1)$$

Im. E_x^r:

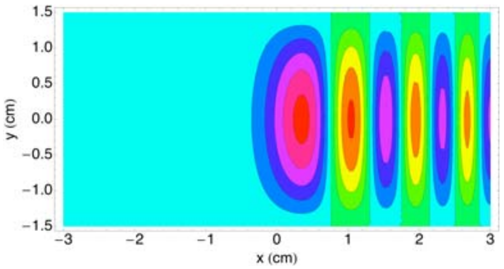


Re. E_x^r:



Beam self-field

Mode pattern:



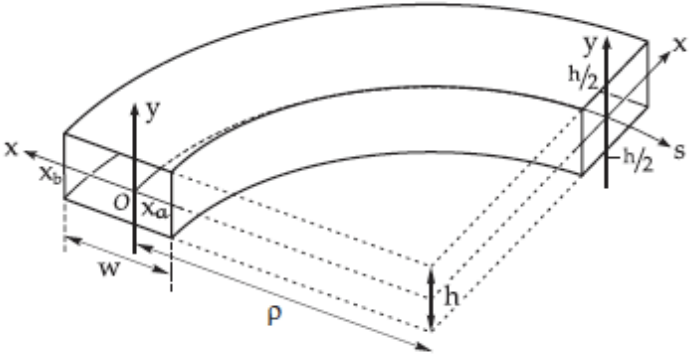
E_x mode: $E_x(x, y) = E_{x0}\mathrm{Ai}\left(k_y^2\kappa^2 - x/\kappa\right)\sin\left[k_y(y + b/2)\right]$

G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

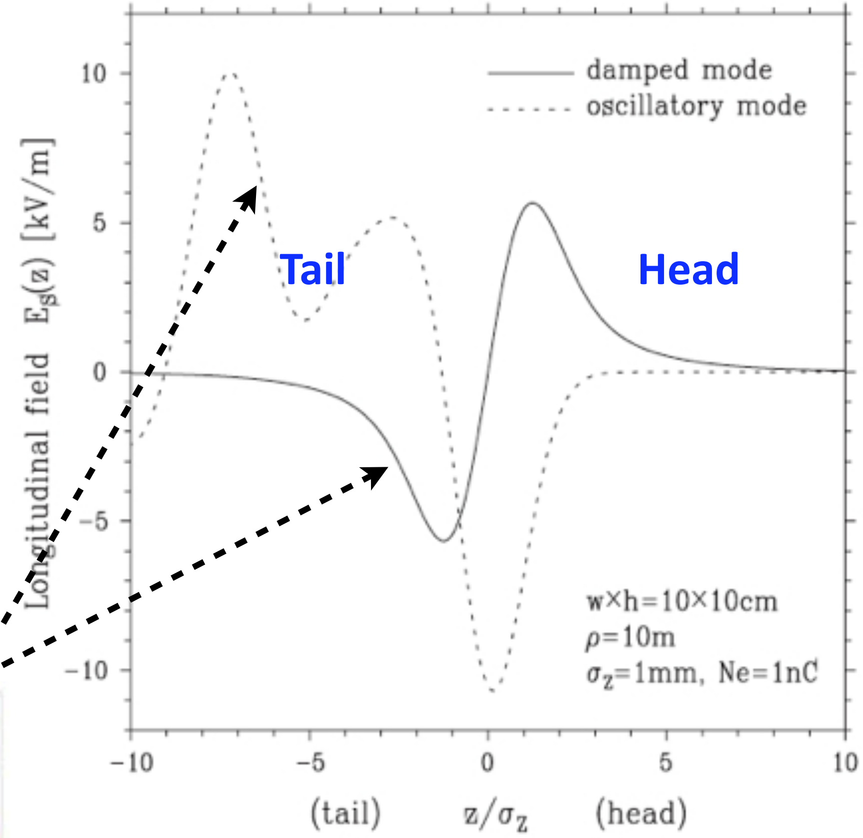
Steady-state CSR

CSR fields can be decomposed to a sum of propagating (oscillatory and trailing) and decaying (damped and overtaking) waves in a toroid waveguide [Agoh (2009)].

Geometry



Long. wakefields for a Gaussian bunch



Re. and Im. Poles

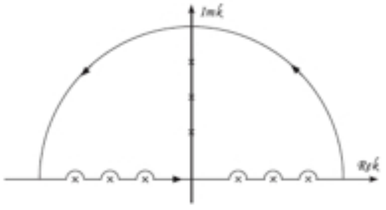


FIG. 7. Contour for $\zeta > 0$.

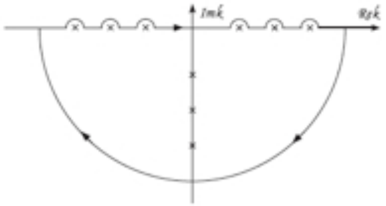
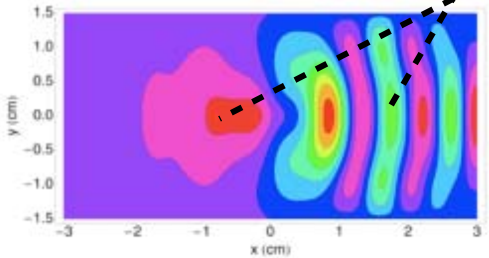


FIG. 8. Contour for $\zeta < 0$.

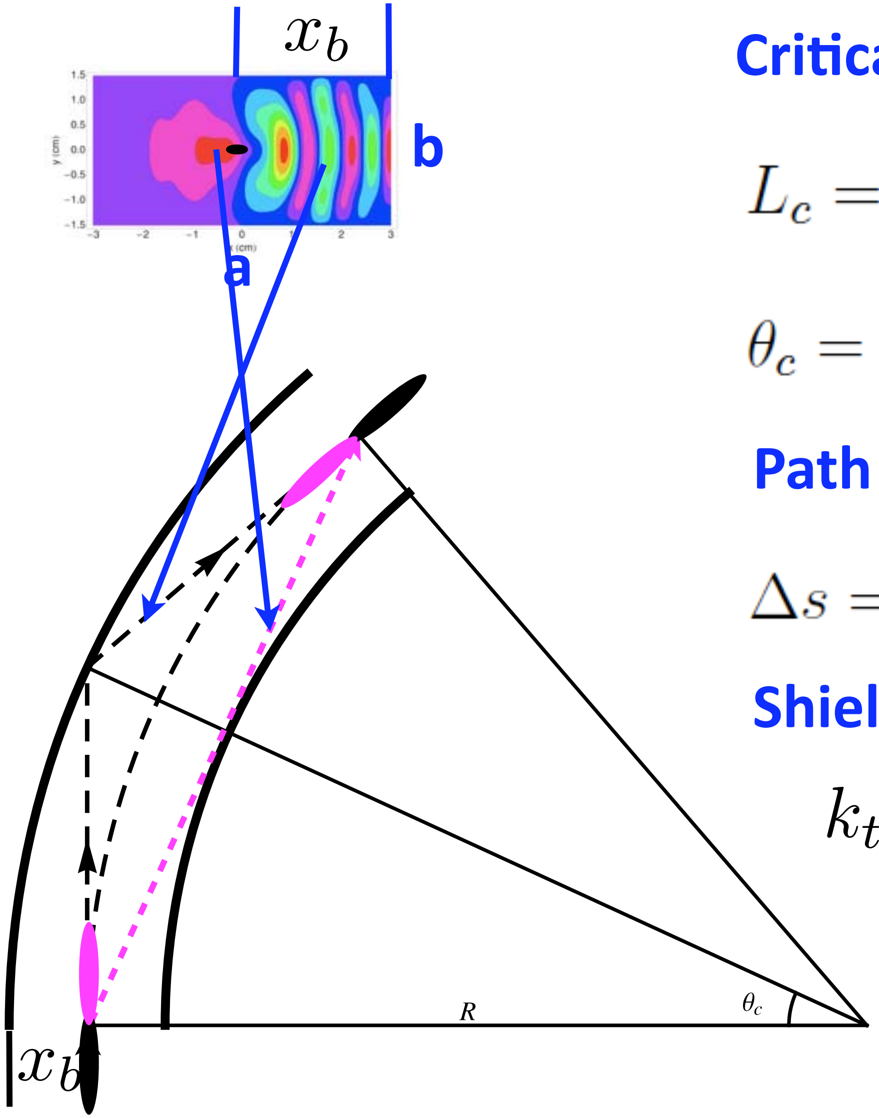


T. Agoh, PRST-AB 12, 094402 (2009).

Geometric model: optical approximation

Side-wall reflection can be approximated by a geometric model

[Derbenev (1995), Carr (2001), Sagan (2009), Oide (2010)]



Critical length (Catch-up distance):

$$L_c = 2R\theta_c \approx 2\sqrt{2Rx_b} \qquad x_b \ll R$$

$$\theta_c = \mathrm{ArcCos}\left(R/(R+x_b)\right) \approx \sqrt{2x_b/R}$$

Path difference:

$$\Delta s = 2R(\mathrm{Tan}(\theta_c) - \theta_c) \approx \frac{4}{3}\sqrt{\frac{2x_b^3}{R}}$$

Shielding threshold:

$$k_{th} = \pi\sqrt{R/b^3}$$

Y. S. Derbenev, et al., TESLA FEL-Report 1995-05 (1995).
G. L. Carr, et al., PAC'01, p. 377 (2001).
D. Sagan, et al., PRST-AB 12, 040703 (2009).
K. Oide, Talk at CSR mini-workshop, Nov. 08, 2010.
D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

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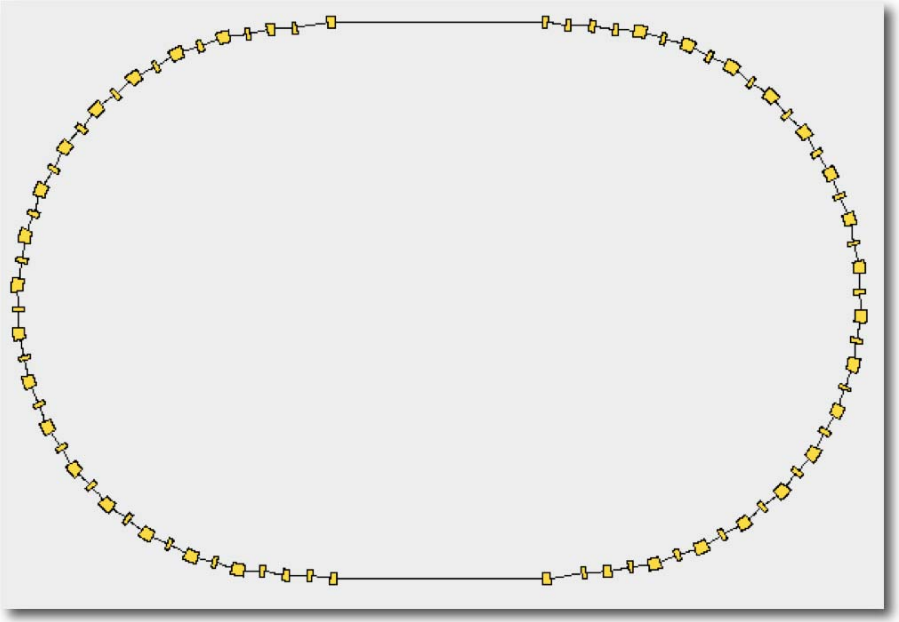
Application to SuperKEKB DR

SuperKEKB damping ring: multi-bend interference

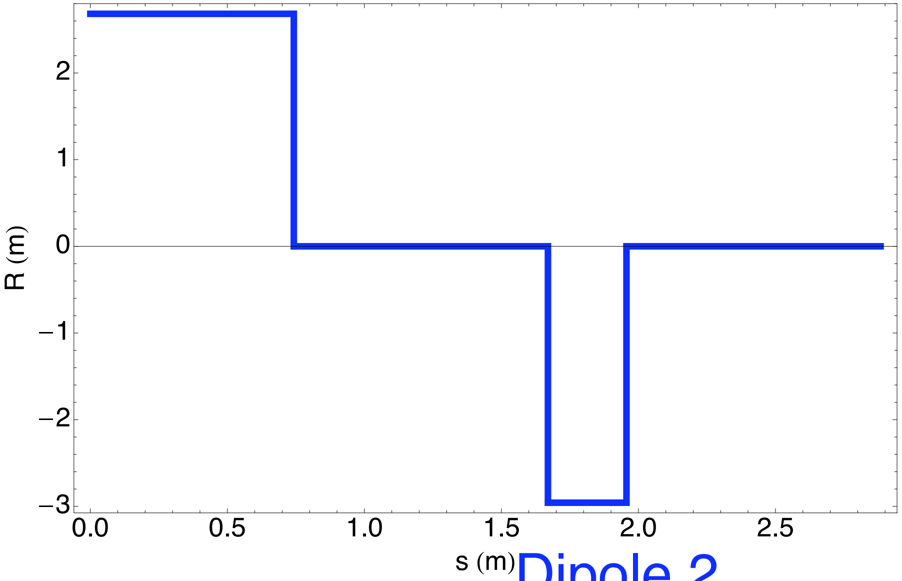
a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends)
L_{drift}=0.9 m, N_{cell}=32

The vacuum chamber is curved along the beam orbit



Layout

Dipole 1



Dipole 2

Reverse bends (1 cell)

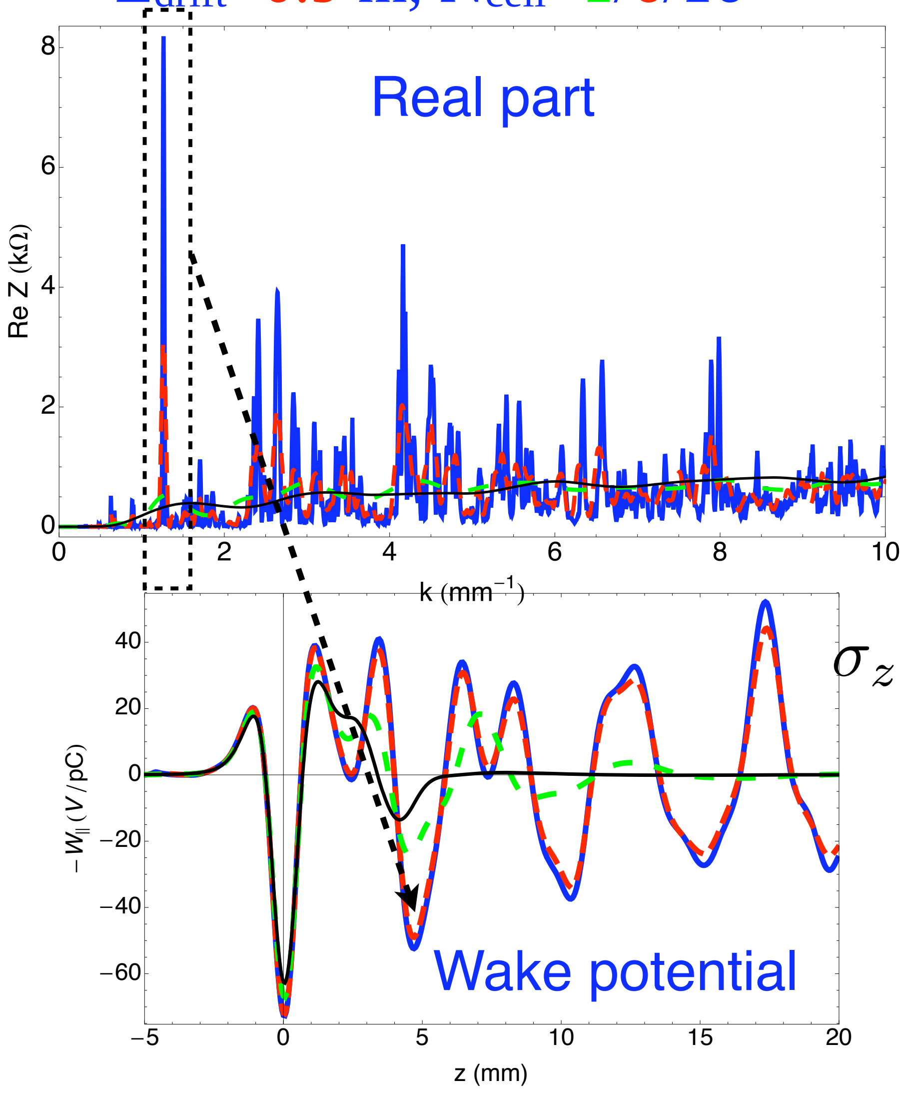
M. Kikuchi, et al., IPAC'10, p. 1641 (2010)

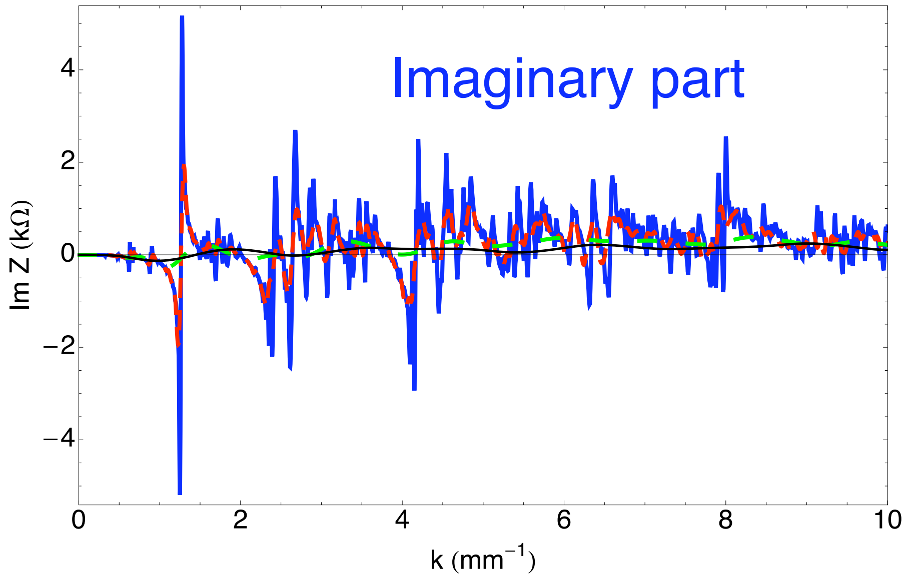
Application to SuperKEKB DR (cont'd)

SuperKEKB damping ring (one arc section) (Perfect conducting wall)

a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends)

L_{drift}=0.9 m, N_{cell}=1/6/16





$$\sigma_z = 0.5\ \mathrm{mm}$$

Blue solid lines: 16 cells

Red dashed lines: 6 cells

Green dotted lines: 1 cell

Black solid lines: single-bend

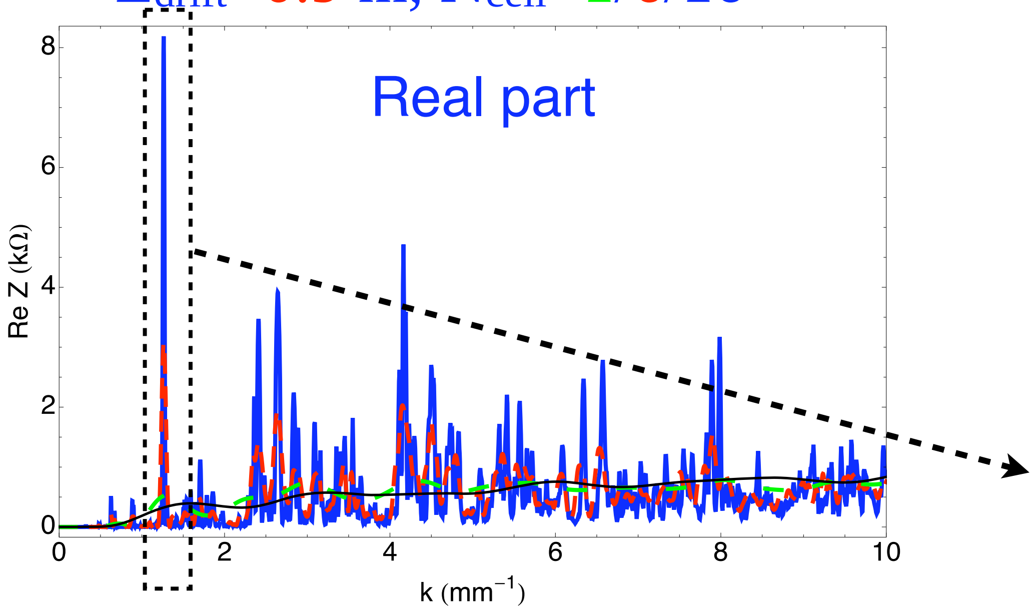
D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

Application to SuperKEKB DR (cont'd)

SuperKEKB damping ring (one arc section) (Perfect conducting wall)

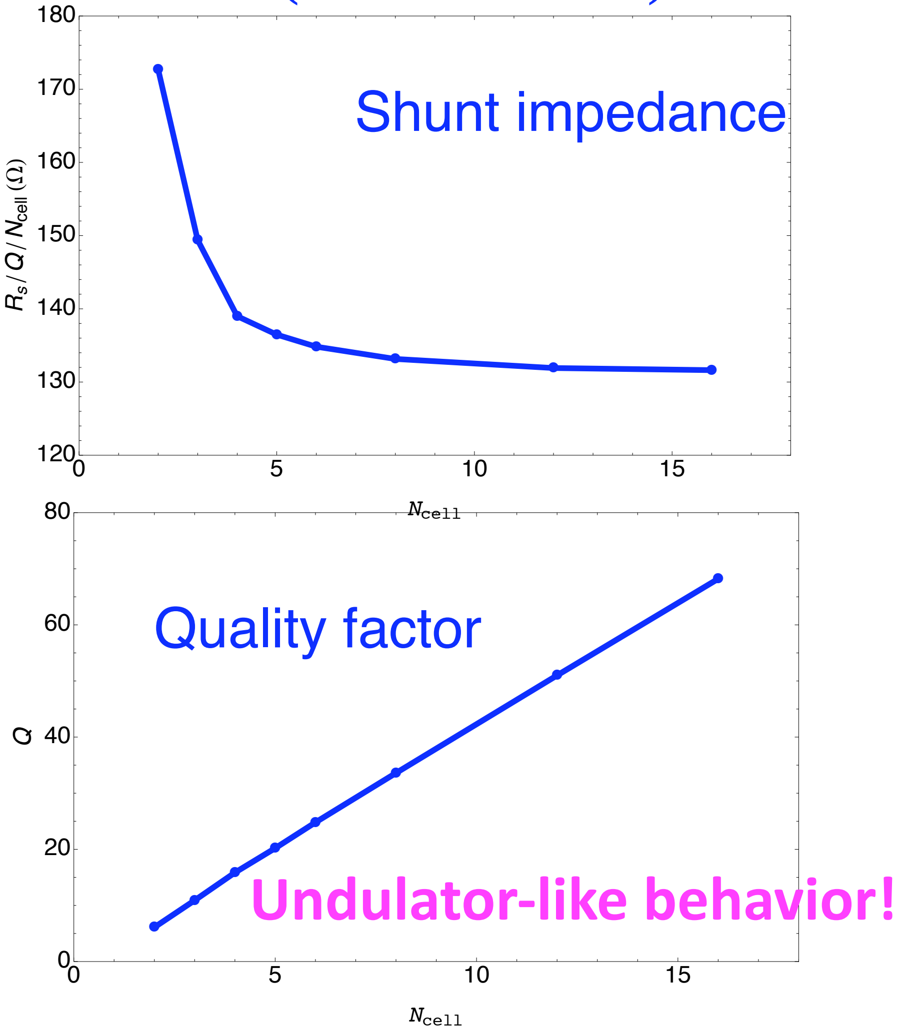
a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends)

L_{drift}=0.9 m, N_{cell}=1/6/16



Resonator model:

$$Z_{\parallel}(k) = \frac{R_s}{1 + iQ(k_r/k - k/k_r)}$$



Undulator-like behavior!

D. Zhou, et al., to be published in Jpn. J. Appl. Phys..; H. Ikeda, et al., THPZ021, IPAC'11
N. Iida, et al., THYA01, IPAC'11

SuperKEKB DR: Microwave instability

SuperKEKB DR: CSR instability threshold

Keil-Schnell-Boussard criterion: $$\left|\frac{Z_{\parallel}}{n}\right| < FZ_0\frac{\gamma\alpha_p\sigma_\delta^2\sigma_z}{N_0 r_e}$$

Condition for K-S-B criterion: broad-band impedance
K.Y. Ng (1986) proposed a criterion for narrow-band impedance:

$$\left|\frac{\sqrt{2\pi}k_0\sigma_z}{4}\frac{R_s}{Q}\right| < FZ_0\frac{\gamma\alpha_p\sigma_\delta^2\sigma_z}{N_0 r_e}$$

Machine parameters: $$E = 1.1 \text{ GeV},\ \alpha_p = 0.0141,\ \sigma_\delta = 5.5\times 10^{-4},$$ $$\sigma_z = 7.74 \text{ mm},\ N_0 = 5 * 10^{10}$$

For SuperKEKB DR, the K-S-B criterion give a threshold of $|Z_{\parallel}/n| <$ 0.24 Ω. But when applying Ng's criterion to the sharp peak at k_r=1.264 mm^{-1}, it gives an impedance of 0.95 Ω.

Conclusion: interfered CSR is important in the SuperKEKB DR.

M. Kikuchi, et al., IPAC'10, p. 1641 (2010)

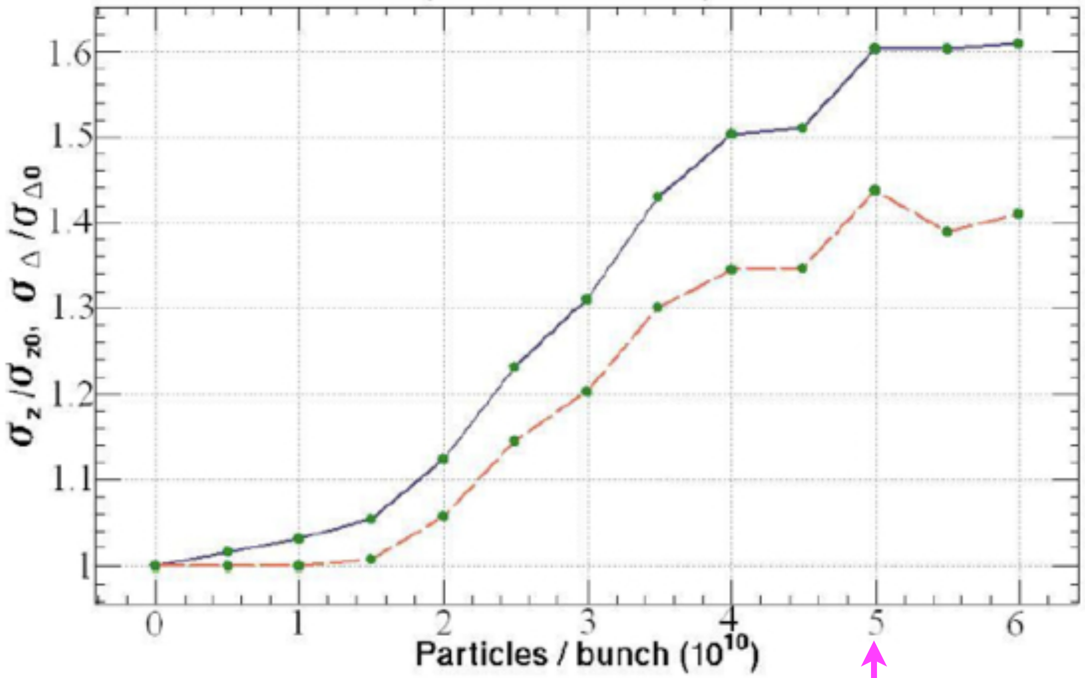
SuperKEKB DR: Microwave instability (cont'd)

SuperKEKB DR (latest version): CSR instability threshold [Cai (2011)]:

$$\chi = \sigma_z \sqrt{\frac{\rho}{h^3}} \approx 2.9 \qquad \rho = 2.7\ \mathrm{m} \qquad h = 24\ \mathrm{mm}$$

$$I_b = 0.5 * \frac{3\sqrt{2}\alpha\gamma\sigma_\delta^2 I_A \sigma_z}{\pi^{3/2} h} = 0.016\ \mathrm{A} \qquad N_{th} = \frac{I_b C}{ec} \approx 4.6 \times 10^{10}$$

SuperKEKB DR: simulations using Vlasov solver [Ikeda (2011)]:



Design

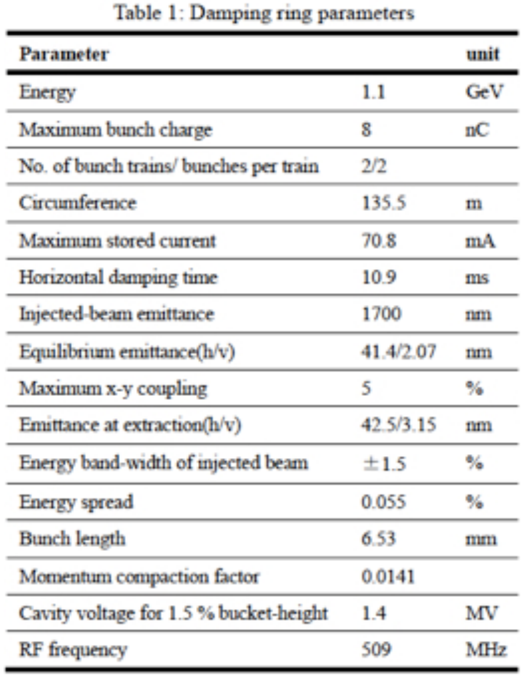


Table 1: Damping ring parameters

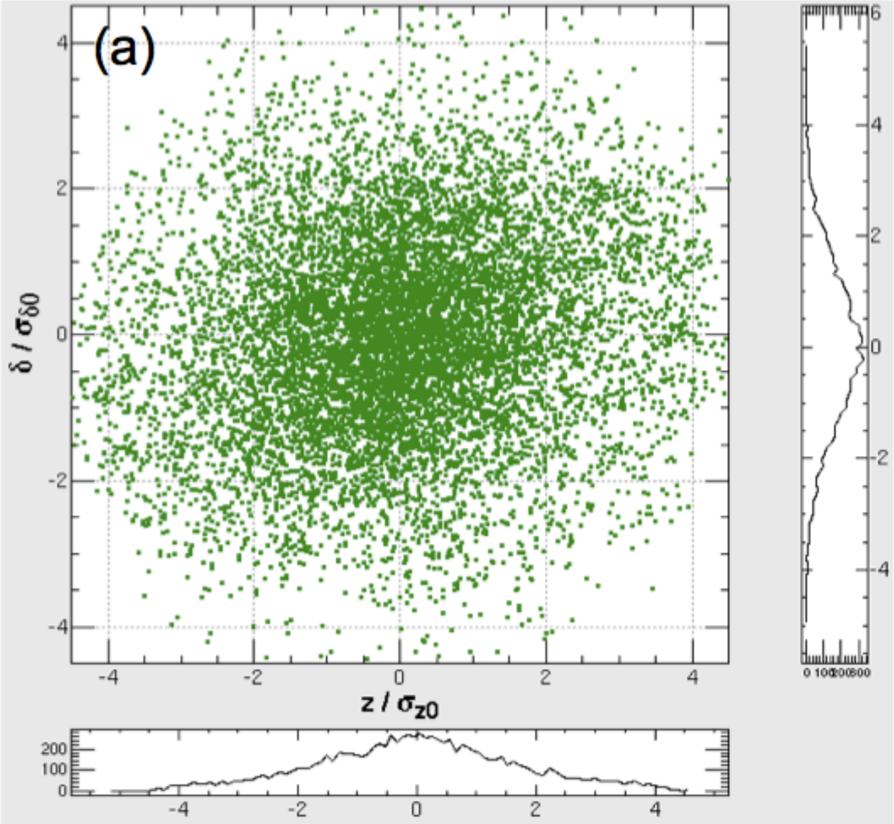
Parameter		unit
Energy	1.1	GeV
Maximum bunch charge	8	nC
No. of bunch trains/ bunches per train	2/2	
Circumference	135.5	m
Maximum stored current	70.8	mA
Horizontal damping time	10.9	ms
Injected-beam emittance	1700	nm
Equilibrium emittance(h/v)	41.4/2.07	nm
Maximum x-y coupling	5	%
Emittance at extraction(h/v)	42.5/3.15	nm
Energy band-width of injected beam	±1.5	%
Energy spread	0.055	%
Bunch length	6.53	mm
Momentum compaction factor	0.0141	
Cavity voltage for 1.5 % bucket-height	1.4	MV
RF frequency	509	MHz

Y. Cai, FRXAA01, IPAC'11 (2011)
H. Ikeda, et al., THPZ021, IPAC'11 (2011)

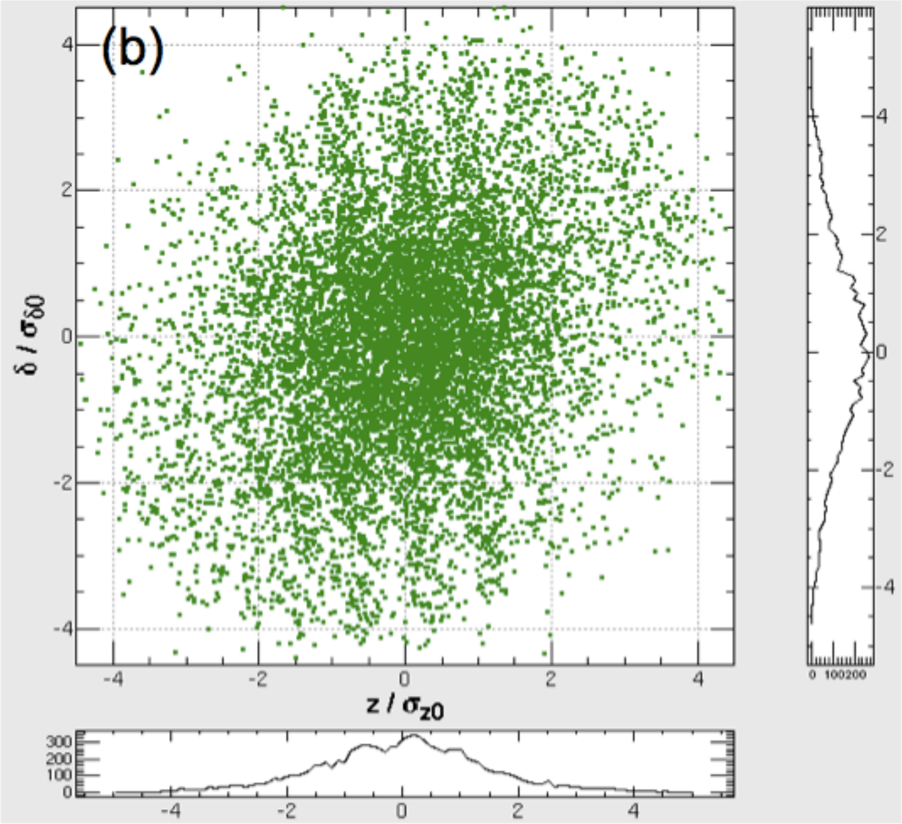
SuperKEKB DR: Microwave instability (cont'd)

SuperKEKB DR: high-freq. modulation was observed in simulations

[Iida (2011)]



Weak modulation



Strong modulation

N. Iida, et al., THYA01, IPAC'11 (2011)

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Application to cERL

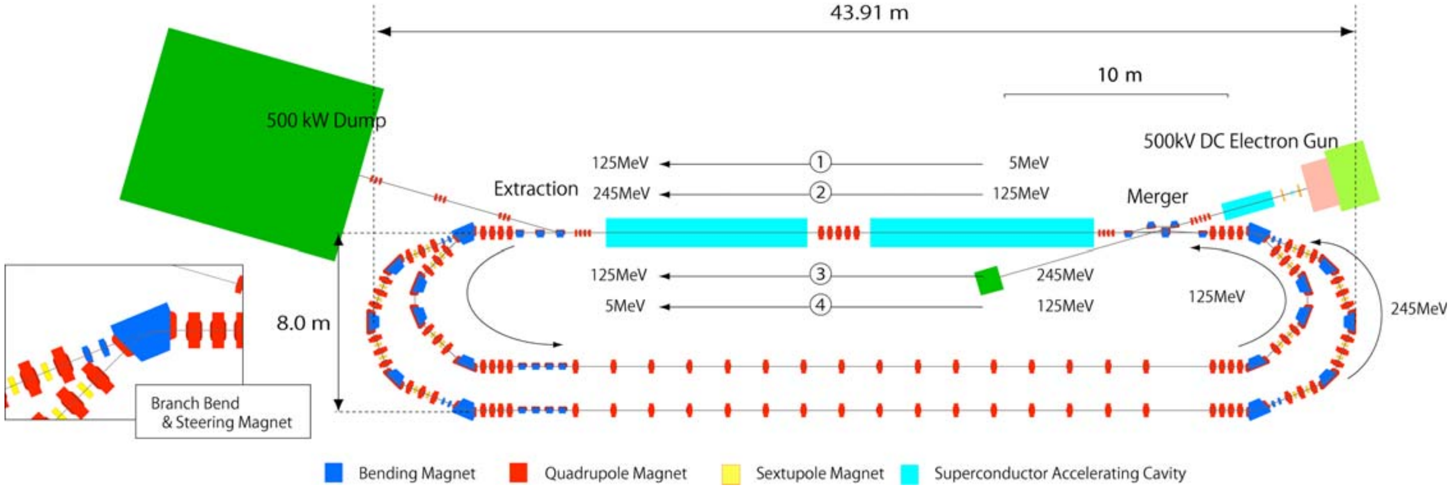
cERL loop: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m
First commissioning: 35 MeV

Injection energy	5- 10 MeV
Full energy	245 MeV
Electron charge	77 pC
Normalized emittance	< 1 mm-mrad
Bunch length	1-3 ps

Main parameters

Multi-bend interference is not important (?)

$$\sigma_z \ll \Delta s = \frac{4}{3}\sqrt{\frac{2x_b^3}{R}} \approx 7.5\ \mathrm{mm}$$



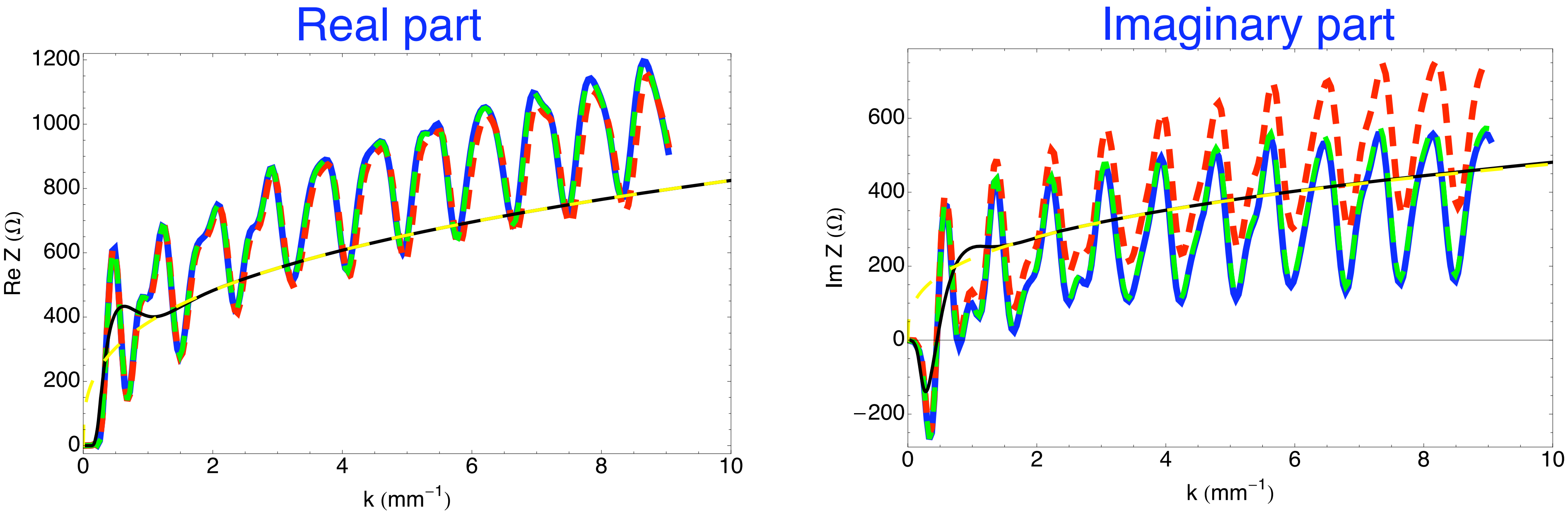
Layout of double loop Compact ERL

Ref. M. Shimada's talk, Tue.@WG2

Application to cERL (cont'd)

Single dipole: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m

Beam line: (Bend + Infinite drift)



$$\sigma_x = 0.2 \text{ mm} \quad \sigma_y = 0.1 \text{ mm}$$

Blue solid lines: $\gamma=\infty$

Red dashed lines: γ=68.5 (E=35 MeV)

Green dashed lines: γ=244.6 (E=125 MeV)

Yellow dashed lines: Steady-state(S-S) Free space model

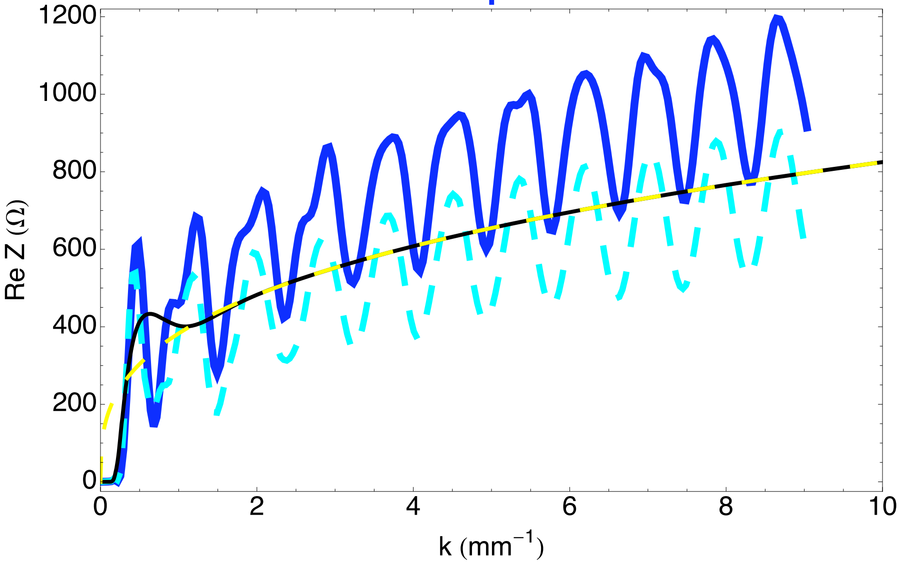
Black solid lines: S-S parallel plates

Application to cERL (cont'd)

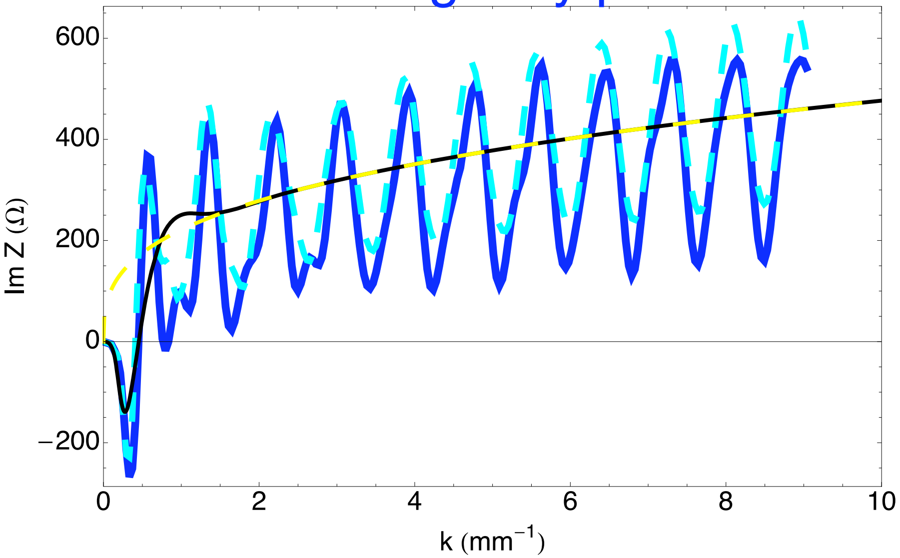
Single dipole: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m

Beam line: (Bend)

Real part



Imaginary part



$$\sigma_x = 0.2\ \mathrm{mm} \quad \sigma_y = 0.1\ \mathrm{mm}$$

Blue solid lines: (Bend + Infinite drift)

Cyan dashed lines: (Bend)

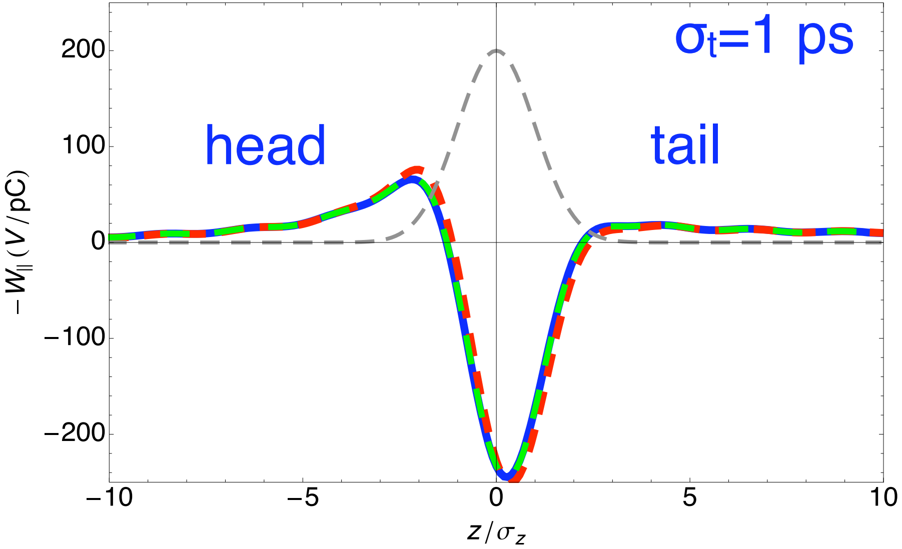
Yellow dashed lines: Steady-state(S-S) Free space model

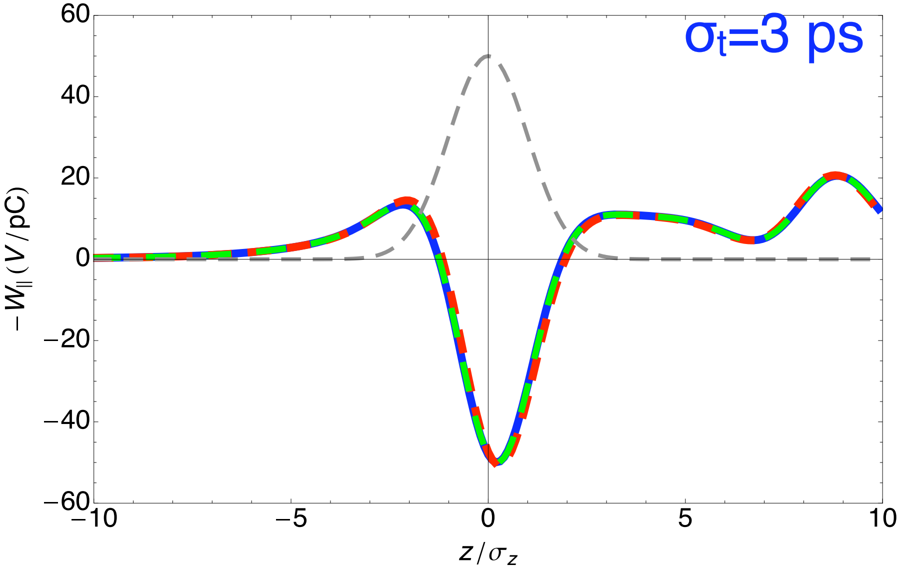
Black solid lines: S-S parallel plates

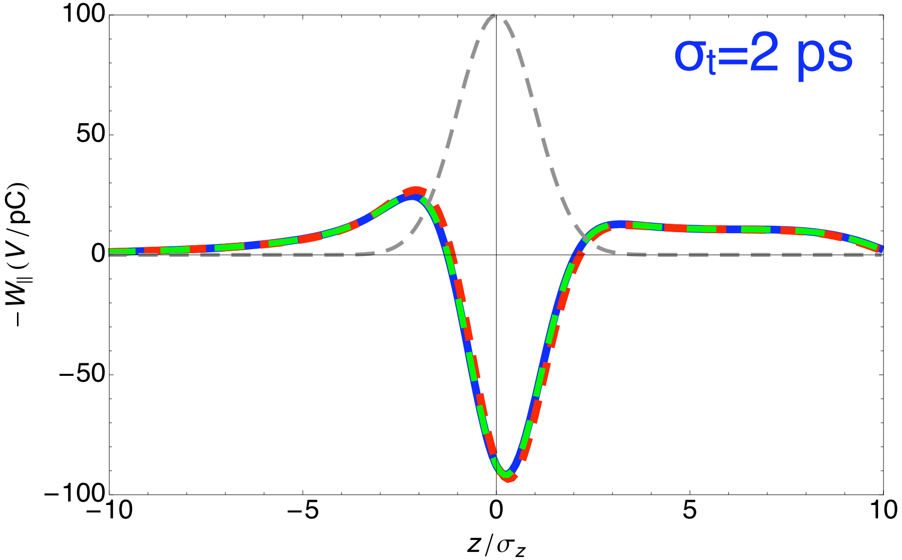
Application to cERL (cont'd)

Single dipole: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m

Beam line: (Bend + Infinite drift)







Blue solid: $\gamma=\infty$

Red dashed: $\gamma=68.5$ (E=35 MeV)

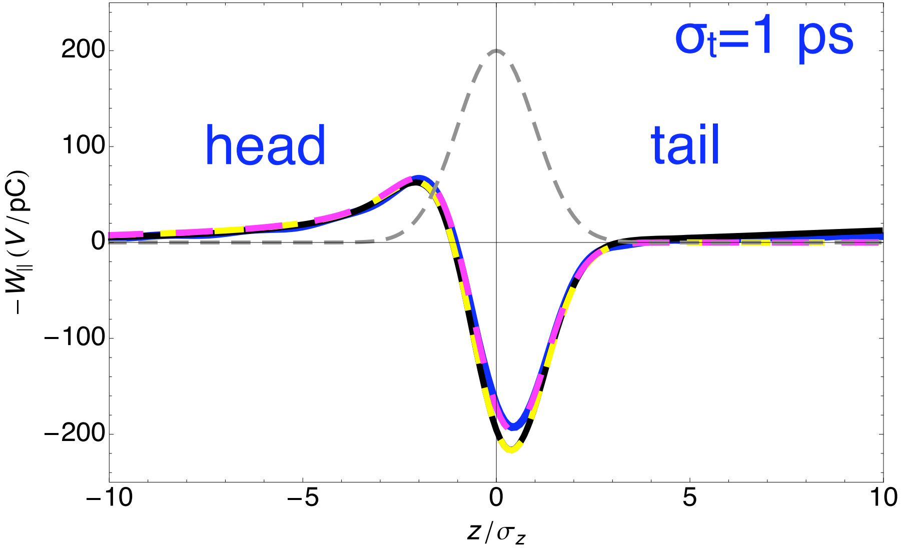
Green dashed: $\gamma=244.6$ (E=125 MeV)

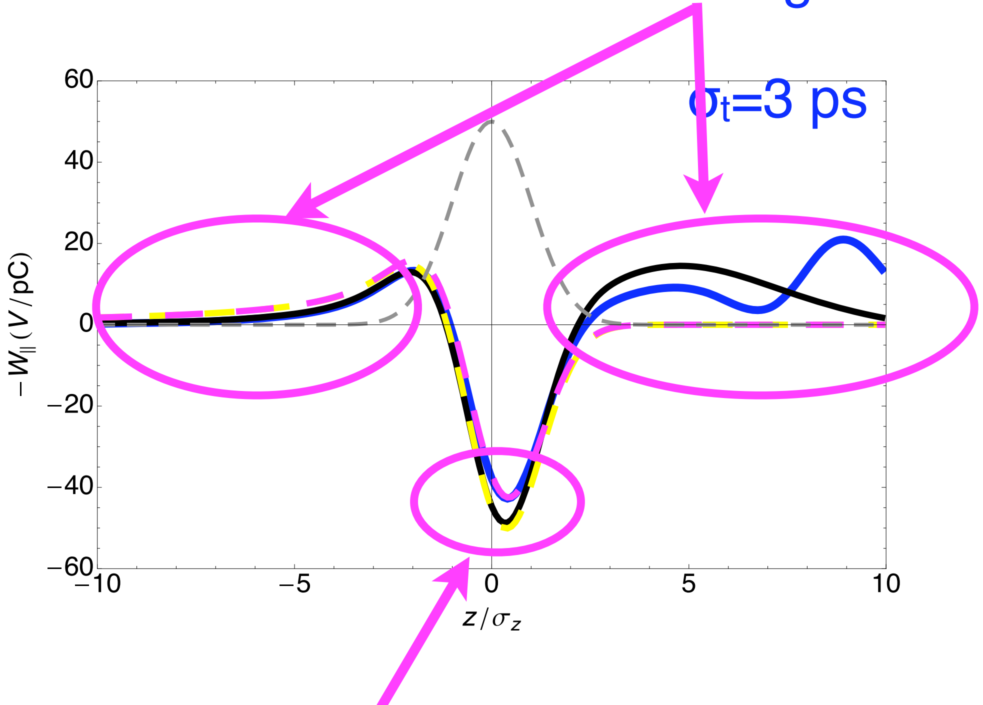
Gray dashed: Gaussian bunch

Application to cERL (cont'd)

Single dipole: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m

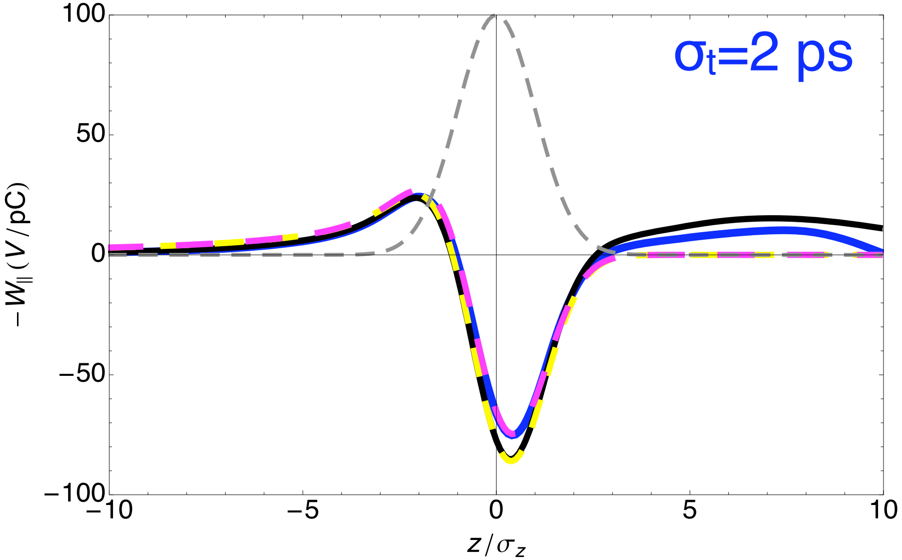
Beam line: (Bend) w/o drift





Shielding

entrance transient



Blue solid: CSRZ (γ=∞)

Magenta dashed: elegant 1D model

Yellow dashed: S-S Free-space model

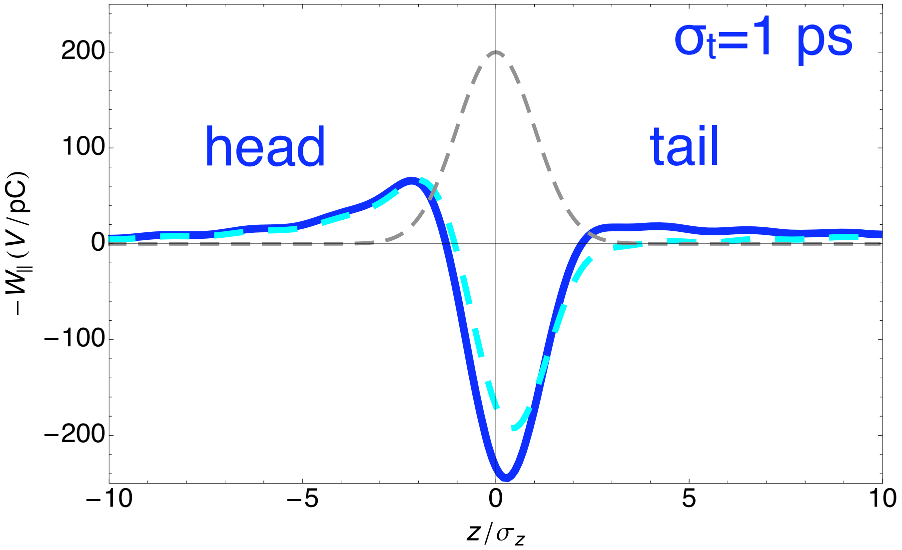
Black solid: S-S parallel plates

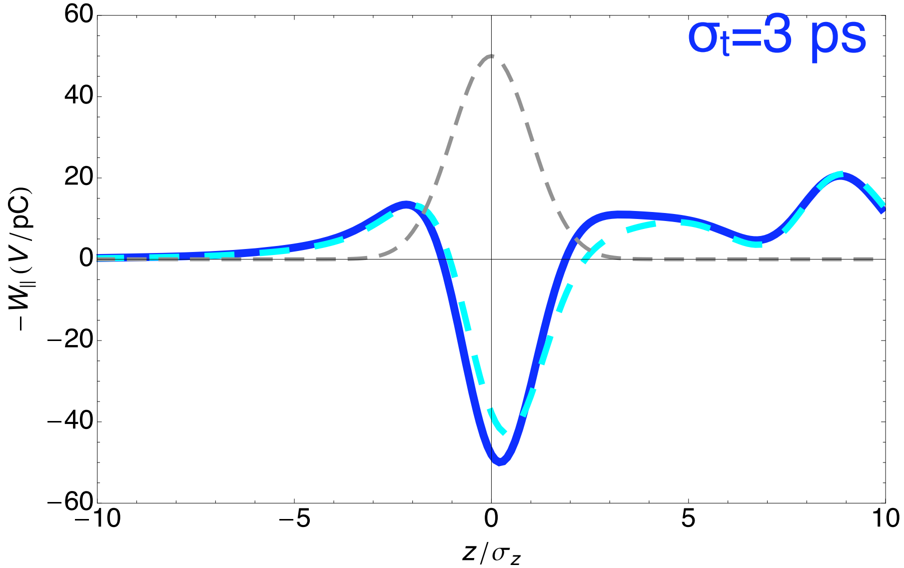
Gray dashed: Gaussian bunch

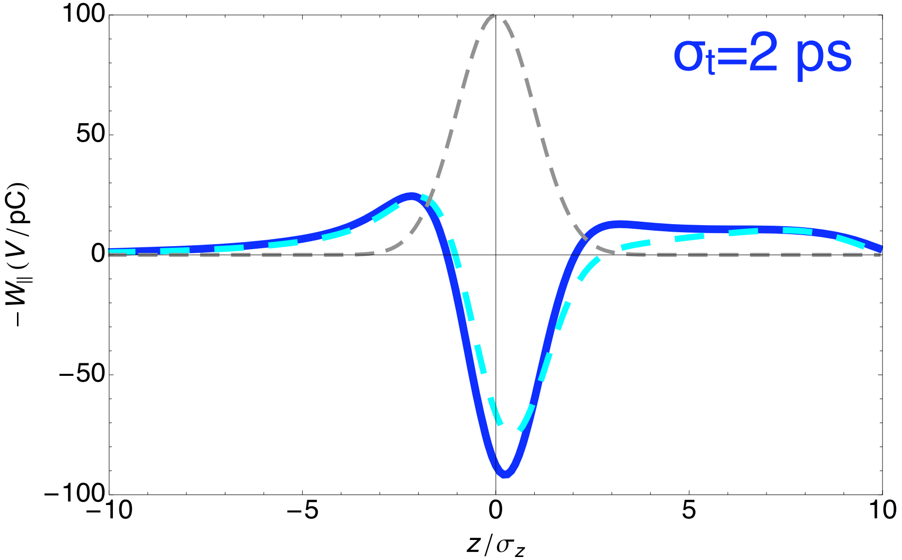
Application to cERL (cont'd)

Single dipole: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m

Effect of drift chamber







Blue solid: w/ drift (CSRZ, γ=∞)

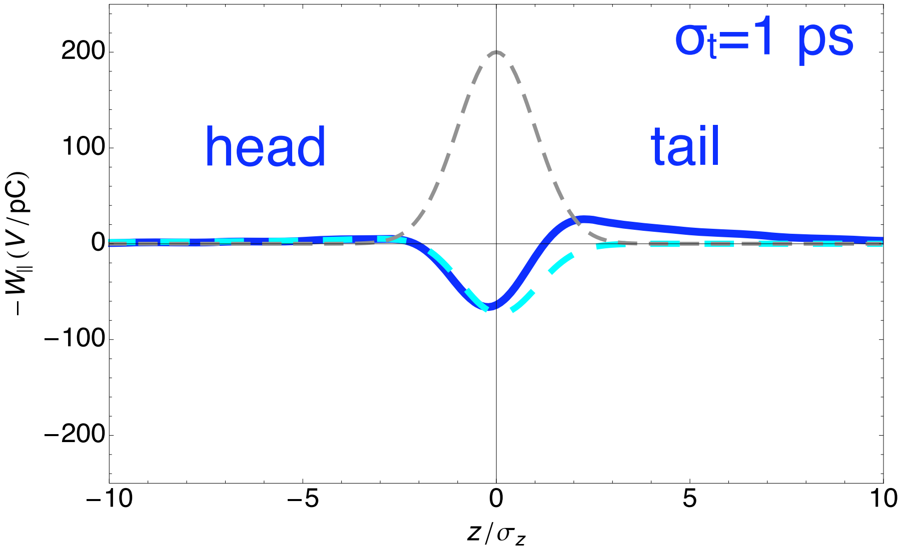
Cyan dashed: w/o drift (CSRZ, γ=∞)

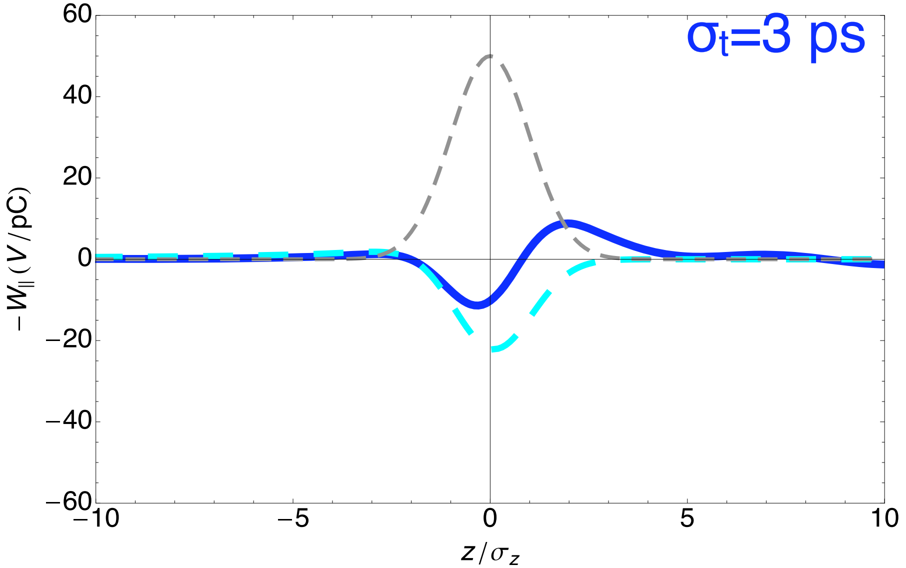
Gray dashed: Gaussian bunch

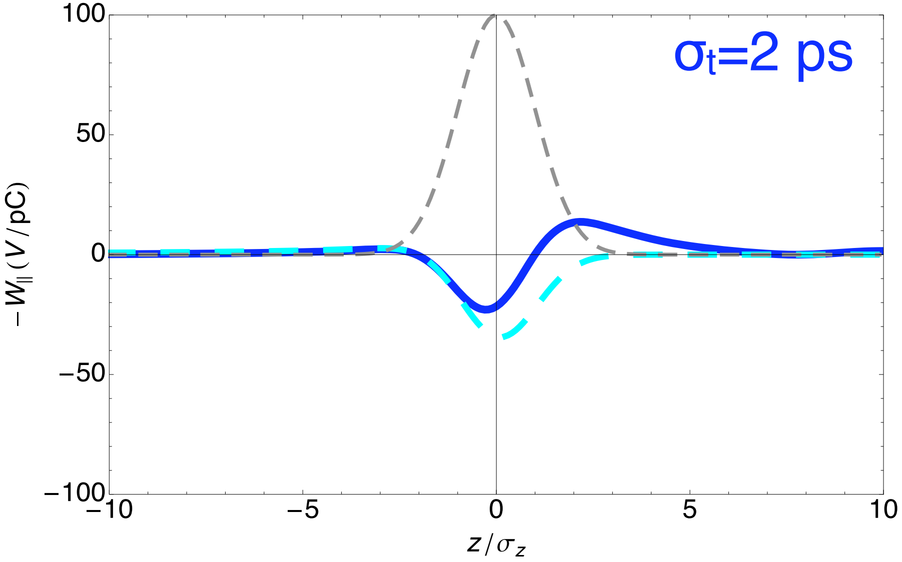
Application to cERL (cont'd)

Single dipole: a/b=50/50 mm, L_{bend}=0.7854 m, R=1 m

Effect of drift chamber







Blue solid: w/ chamber (CSRZ, $\gamma=\infty$)

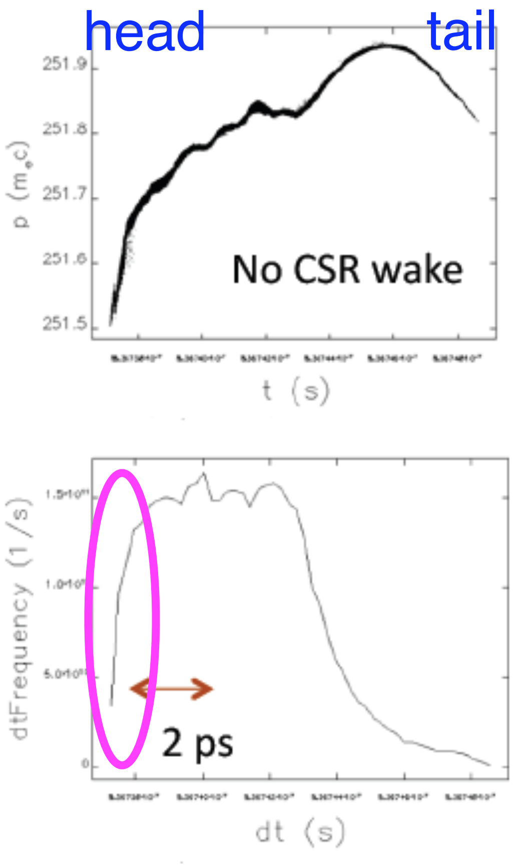
Cyan dashed: free-space model*)

Gray dashed: Gaussian bunch

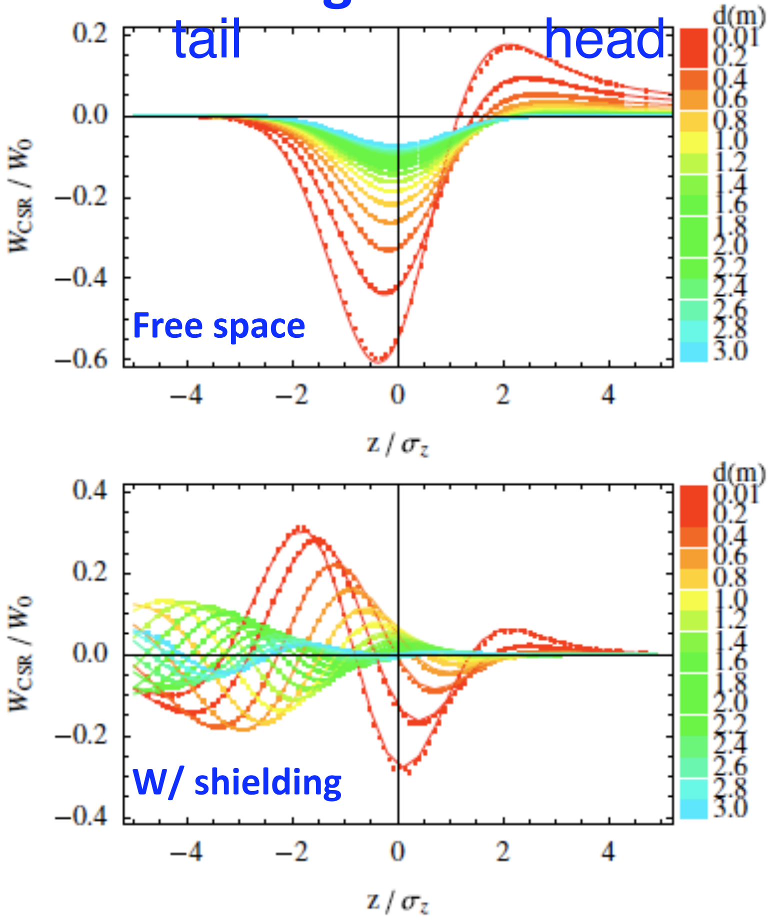
***) G. Stupakov and P. Emma, LCLS-TN-01-12, Dec. 2001.**

Application to cERL (cont'd)

Shielding is ignored in ELEGANT ...



Bmad uses parallel-plates shielding ...



Ref. M. Shimada's talk, ERL2011

D. Sagan, et al., PRST-AB 12, 040703 (2009).

Application to cERL (cont'd)

The popular 1D model for CSR without shielding ...

M. Borland, PRST-AB 4, 070701 (2001)

$$\frac{\partial W_{\parallel}(z,s)}{\partial s} = T_1(z,R,s) + T_2(z,R,s) \qquad z_L = \frac{s^3}{24R^2}$$

$$T_1(z,R,s) = K \int_{z-z_L}^{z} \frac{d\lambda}{dz'} \left(\frac{1}{z-z'}\right)^{1/3} dz' \qquad T_2 = K \frac{\lambda(z-z_L) - \lambda(z-4z_L)}{z_L^{1/3}}$$

An alternative 1D model for CSR with shielding ...

$$Z(k) = -\frac{1}{q}\int_0^{\infty} E_s(x_c, y_c)\,ds \quad \longrightarrow \quad \frac{\partial Z_{\parallel}(k,s)}{\partial s} = -\frac{1}{q} E_s(x_c, y_c)$$

Advantages:

1) Transient effects + chamber shielding;
2) High frequency cutoff;
3) No need for derivative of charge density;
4) Drift CSR (To benchmark with Bmad) ...

Outline

1. Introduction to CSRZ code
2. Field dynamics of CSR
3. Application to SuperKEKB DR
4. Application to cERL@KEK
5. Summary

Summary (1)

From CSRZ:

1. CSR fields can be decomposed to a sum of radiation fields (propagating modes) and beam self-fields (decaying modes) [a proof to T. Agoh's theory (PRST-AB 12, 094402 (2009))].

2. Multi-bend CSR interference appears in small storage rings and may play a role in microwave instability.

To cERL loop (tentative conclusions):

1. Chamber shielding causes remarkable energy kick to the tailing particles in the cases of σ_t>1 ps.

2. Free-space model for drift CSR wake over-estimates the energy kick in the cases of σ_t>1 ps.

3. Longitudinal space-charge effect ($1/\gamma^2$ term) is a concern at E=35 MeV and σ_t<1 ps.

Summary (2)

To do list:

1. **Benchmark of elegant (free-space), Bmad (parallel-plates) and CSRZ (closed chamber): focus on drift CSR, also multi-bend interference.**

2. **Alternative 1D model for CSR: use CSR impedance directly.**

3. **Validity of parabolic equation for CSR calculation**

4. **Macro-particle tracking?**

Thank you!