Study of Resistive-Wall Beam Breakup

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Motivation

- In storage rings such as synchrotron light sources, transverse coupled-bunch instabilities due to long-range resistive-wall wake-fields as well as other instabilities due to RF cavities and ion trapping are sometimes observed at high currents against radiation damping. It is expected that transverse multi-bunch resistive-wall beam breakup(RWBBU) can be a serious problem in high-current ERLs.
- RWBBU has not been hardly studied, while BBU due to HOMs of superconducting cavities has been intensively investigated.
- RWBBU should be fully understood and cures for the BBU must be surveyed and studied to incorporate them into ERL designs.

Resistive-wall wake

Transverse wake function for a round resistive pipe

$$W_\perp = -\frac{cL}{\pi b^3 z^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}} \qquad \left(\sqrt[3]{\frac{b^2}{\sigma_c Z_0}} << z << b^2\sigma_c Z_0\right)$$

L: pipe length b: pipe radius σ_c: electric conductivity

Z_0: vacuum impedance (376.73 Ω) z: distance from wake source

Kick by wake of a preceding bunch with position y & distance z

$$\Delta\theta_y = -\frac{e^2 N}{E} W_\perp \cdot y = \frac{e^2 N}{E}\cdot\frac{cL}{\pi b^3 z^{1/2}}\sqrt{\frac{Z_0}{\pi\sigma_c}}\cdot y$$

N: Number of electrons in bunch E: Beam energy

- Multi-bunch beam easily cumulates wakefields. ($W_\perp \propto z^{-1/2}$)
- Effects of wake depend on pipe characteristics. ($W_\perp \propto b^{-3}\,\sigma_c^{-1/2}\,L$)

Equation of motion

Equation of motion for the *M*-th bunch in a pipe with external focusing

$$y_M''(s) + k_y^2 y_M(s) = \sum_{N=0}^{M-1} S(M-N)\, y_N(s)$$

$$S(M) = \frac{a}{\sqrt{M}}, \quad a \equiv \frac{e^2 N}{E} \cdot \frac{c}{\pi b^3 (c\tau_B)^{1/2}} \sqrt{\frac{Z_0}{\pi \sigma_c}} = \frac{4 I_B}{I_A} \frac{\delta_{skin}}{b^3}$$

$$I_B = \frac{eN}{\tau_B}, \quad I_A = \frac{4\pi\varepsilon_0 m c^3 \gamma}{e}, \quad \delta_{skin} = \sqrt{\frac{\tau_B}{\pi \mu_0 \sigma_c}}$$

τ_B: time separation between bunches k_y: external focusing

s: longitudinal location in resistive-wall pipe

(s=0, L at the entrance and exit of the pipe)

Asymptotic expression for $M \to \infty$ ($M \sim t/\tau_B$)

(J. M. Wang and J. Wu, PRST-AB 7, 034402(2004))

Asymptotic expression (1)

All the injected bunches have the same offset at entrance (injection error)

$\rightarrow\ y_M(0) = y_{00} \neq 0, \quad y_M'(0) = 0 \quad (\forall M)$

(A) No focusing case

$$y_M(s) = \frac{y_{00}}{\sqrt{2\pi}} \left(\frac{t_{NF}}{t} \right)^{1/10} \cdot exp\left[\left(\frac{t}{t_{NF}} \right)^{1/5} \right]$$

$$t \cong M\tau_B \qquad t_{NF} \equiv \frac{\tau_B}{4\pi} \left(\frac{4}{5} \right)^5 \frac{1}{a^2 s^4}$$

(B) Strong focusing case

$$y_M(s) = \frac{y_{00}}{\sqrt{2\pi}} \left(\frac{t_{SF}}{t} \right)^{1/6} \cdot exp\left[\left(\frac{t}{t_{SF}} \right)^{1/3} \right] \cdot cos\left[\sqrt{3} \left(\frac{t}{t_{SF}} \right)^{1/3} - k_y s - \frac{\pi}{6} \right]$$

$$t \cong M\tau_B \qquad t_{SF} \equiv \frac{16 k_y^2 \tau_B}{\pi} \left(\frac{2}{3} \right)^3 \frac{1}{a^2 s^2}$$

Asymptotic expression (2)

Requirements for asymptotic expressions

(A) No Focusing case

$$\alpha_1^{4/5} \equiv \left[\frac{\sqrt{a\sqrt{\pi}}L}{4(M+1)}\right]^{4/5} << 1, \quad \chi_{NF} \equiv \frac{a\sqrt{\pi}}{k_y^{\,2}\alpha_1^{\,2/5}} >> 1, \quad \frac{Mc\tau_B}{L} >> 1, \quad t \approx M\tau_B >> t_{NF}$$

(B) Strong focusing case

$$\alpha_2^{2/3} \equiv \left[\frac{\sqrt{\pi}aL}{4(M+1)}\right]^{4/5} << 1, \quad \chi_{SF} \equiv \frac{k_y^{\,2}\alpha_2^{\,1/3}}{a\sqrt{\pi}} >> 1, \quad \frac{Mc\tau_B}{L} >> 1, \quad t \approx M\tau_B >> t_{SF}$$

Asymptotic expressions are valid

only for limited parameter ranges and initial conditions.

→ Simulation of resistive-wall beam breakup is needed and useful.

RWBBU simulation program

Features of RWBBU simulation program

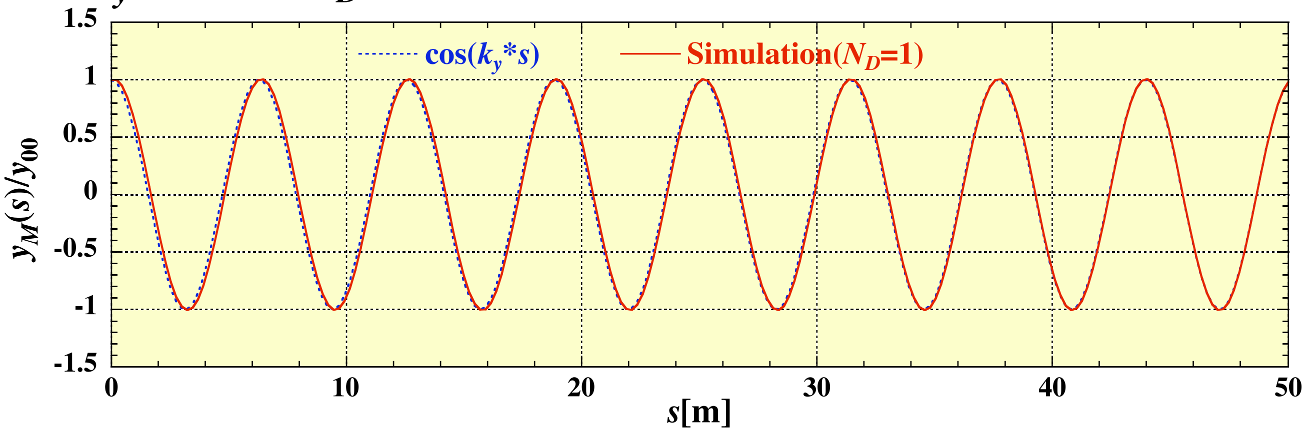
- Transverse long-range resistive-wall wake
- Only lowest mode(dipole)
- Rigid and point-like bunch
- 1-D tracking
- Single pass (no recirculation)
- Round pipes with/without external focusing(or defocusing)
- Non-linear fields not included
- Basic time increment is bunch separation τ_B divided by an integer N_D, τ_B/N_D. (N_D : division number)

Simulation program should be further developed.

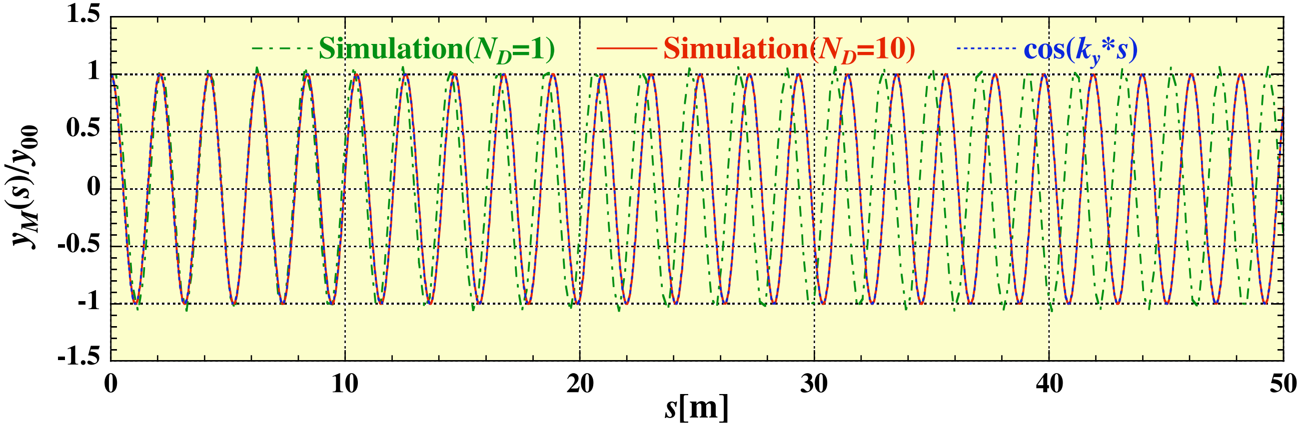
Comparison with analytic solutions (1)

No wake(a=0), Uniform external focusing($k_y \neq 0$)

(a) k_y=1[m^{-1}], N_D=1



(b) k_y=3[m^{-1}], N_D=1, 10



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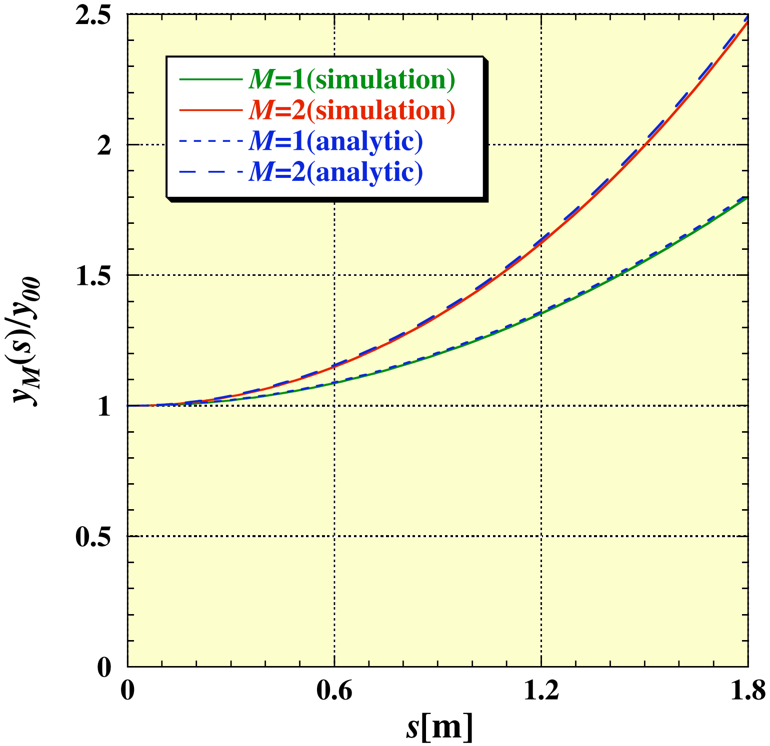
Comparison with analytic solutions (2)

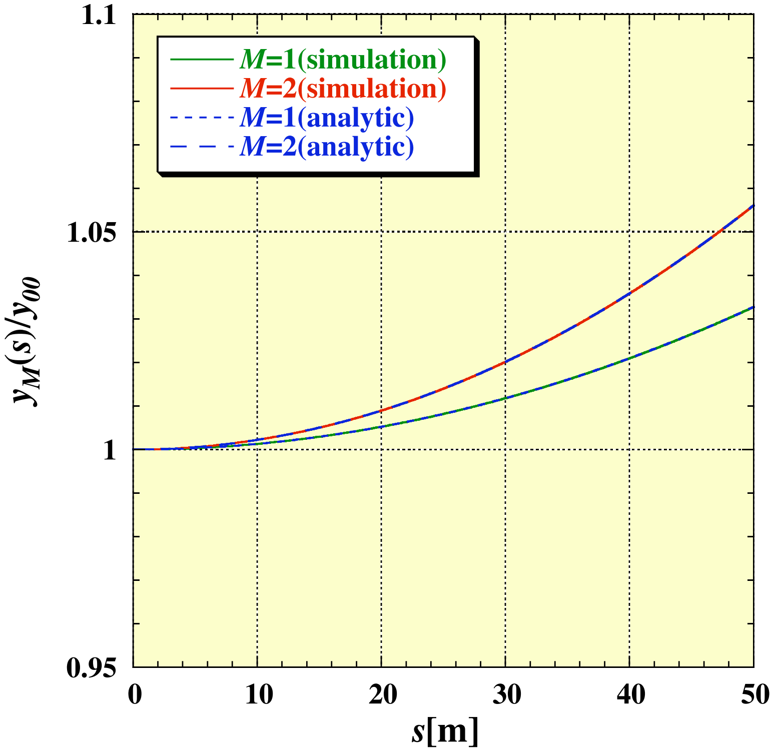
Non-zero wake($a \neq 0$), No external focusing($k_y = 0$)

Analytic solution: $y_1(s)/y_{00} = 1 + \frac{1}{2}as^2, \quad y_2(s)/y_{00} = 1 + \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)as^2 + \frac{1}{24}a^2 s^4$

(1) a=0.5[m^{-2}], L=1.8[m]



(2) a=2.62x10^{-5}[m^{-2}], L=50[m]



Simulation results are consistent with the analytic exact solutions.

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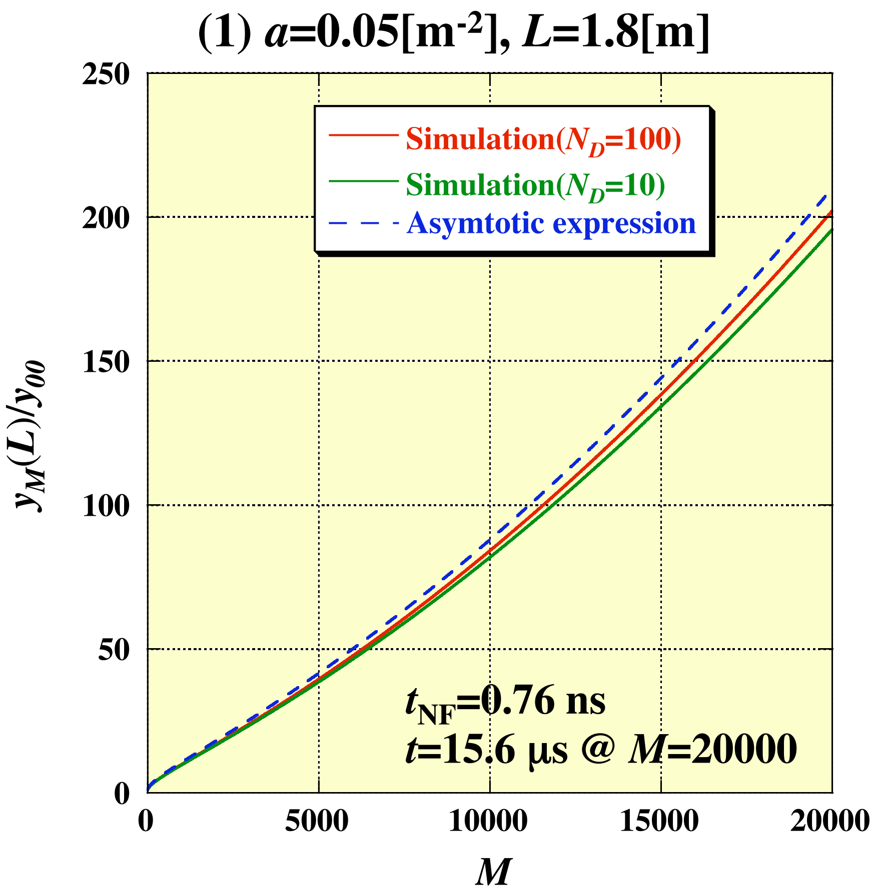
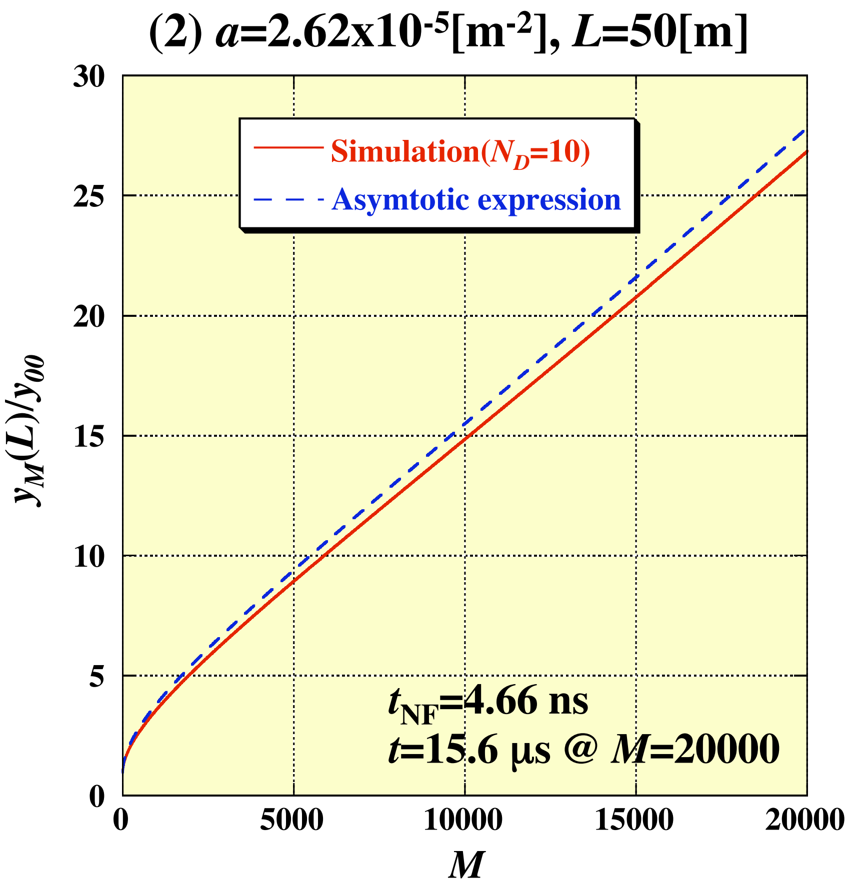
Comparison with asymptotic expression (1)

No focusing case(k_y=0)





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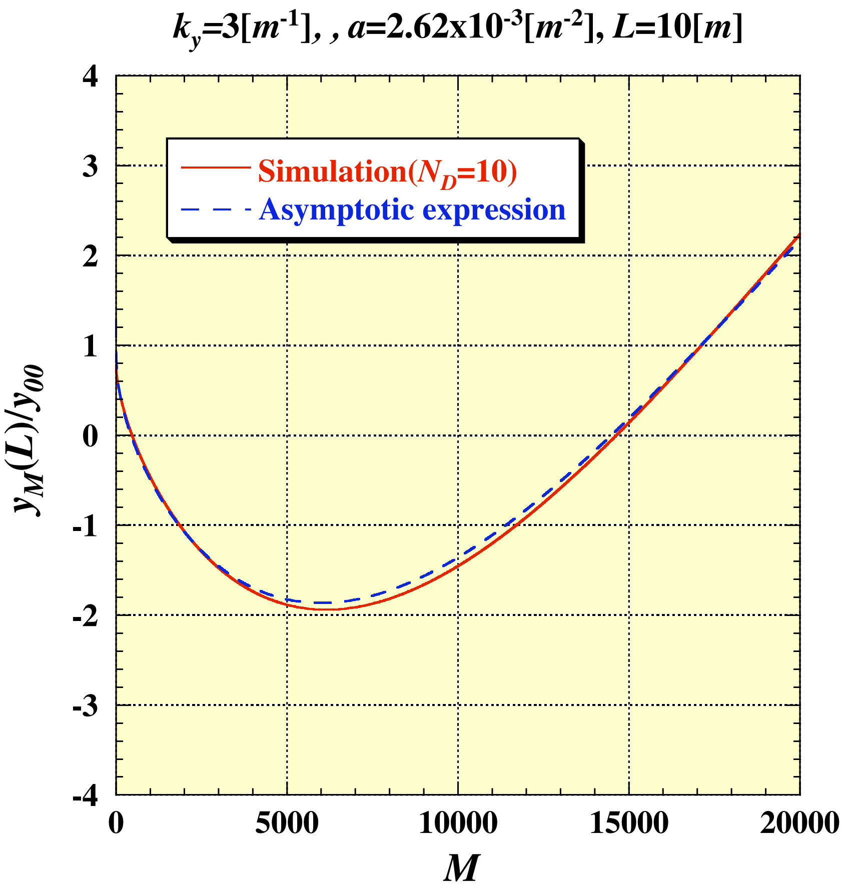
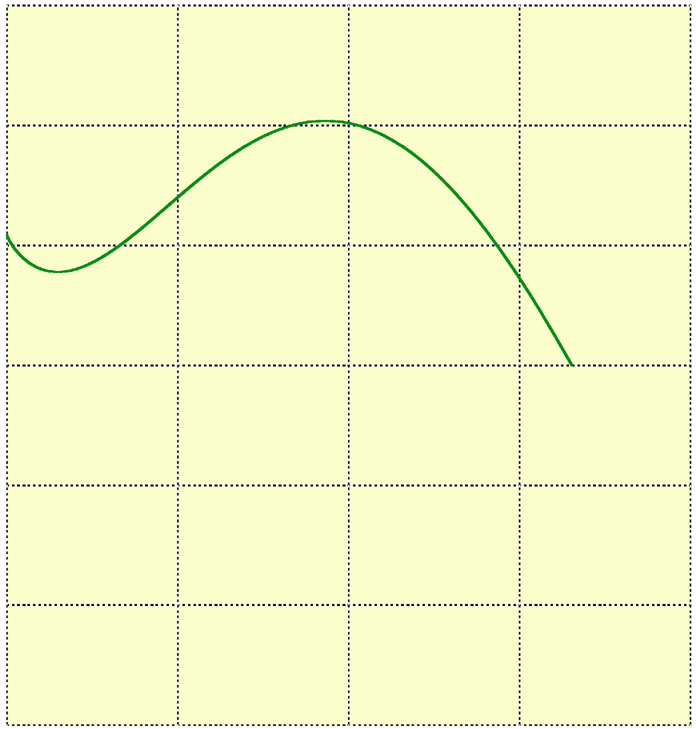
Comparison with asymptotic expression (2)

Strong focusing case

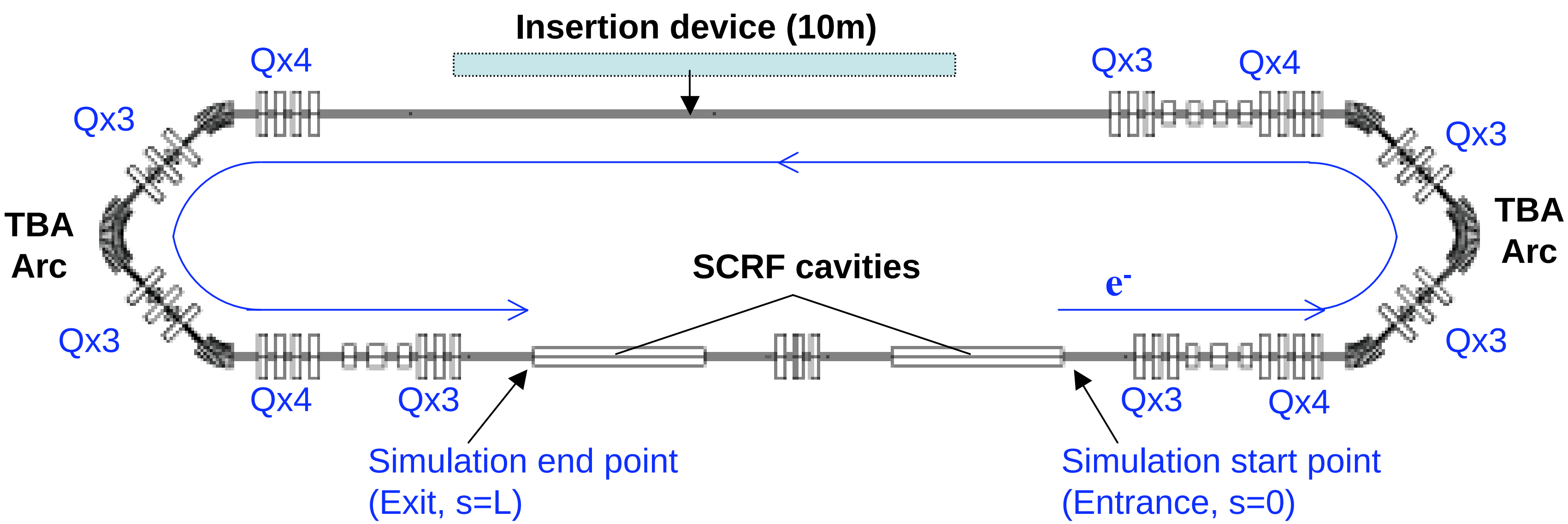




Simulation in ERL (1)

Simulation conditions:

- E = 60 MeV, I_B = 100 mA (uniform bunch filling), τ_B=0.769 ns(f_{RF}=1.3GHz)
  b = 25 mm(Al pipe), L = 56.4 m (total path)
- 1-D tracking in the vertical direction for $t \leq 77.1\ \mu s$ ($M \leq 100000$)
- All the injected bunches have the same position offset at entrance (Injection error).
- Thirty-seven quadrupole magnets are considered.
- Insertion device(ID) : b_{ID}= 10 mm (stainless steel pipe), L_{ID}=10m
- Sextupole magnet fields, orbit correction and alignment errors are not considered.



Layout of test ERL in Japan

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Simulation in ERL (2)

Basic equations for RWBBU simulation

$$y_M{}''(s) + K_y(s) y_M(s) = \sum_{N=1}^{M-1} \frac{a}{\sqrt{M-N}} y_N(s) \quad (M \geq 2)$$

$$y_1{}''(s) + K_y(s) y_1(s) = 0$$

$$\longrightarrow \quad \xi_M{}''(s) + K_y(s) \xi_M(s) = \sum_{N=1}^{M-1} \frac{a}{\sqrt{M-N}} \frac{y_N(s)}{y_1(0)} \quad \left(\xi_M(s) \equiv \frac{y_M(s) - y_1(s)}{y_1(0)} \right)$$

Early stage of RWBBU: $\xi_M(s) << 1$

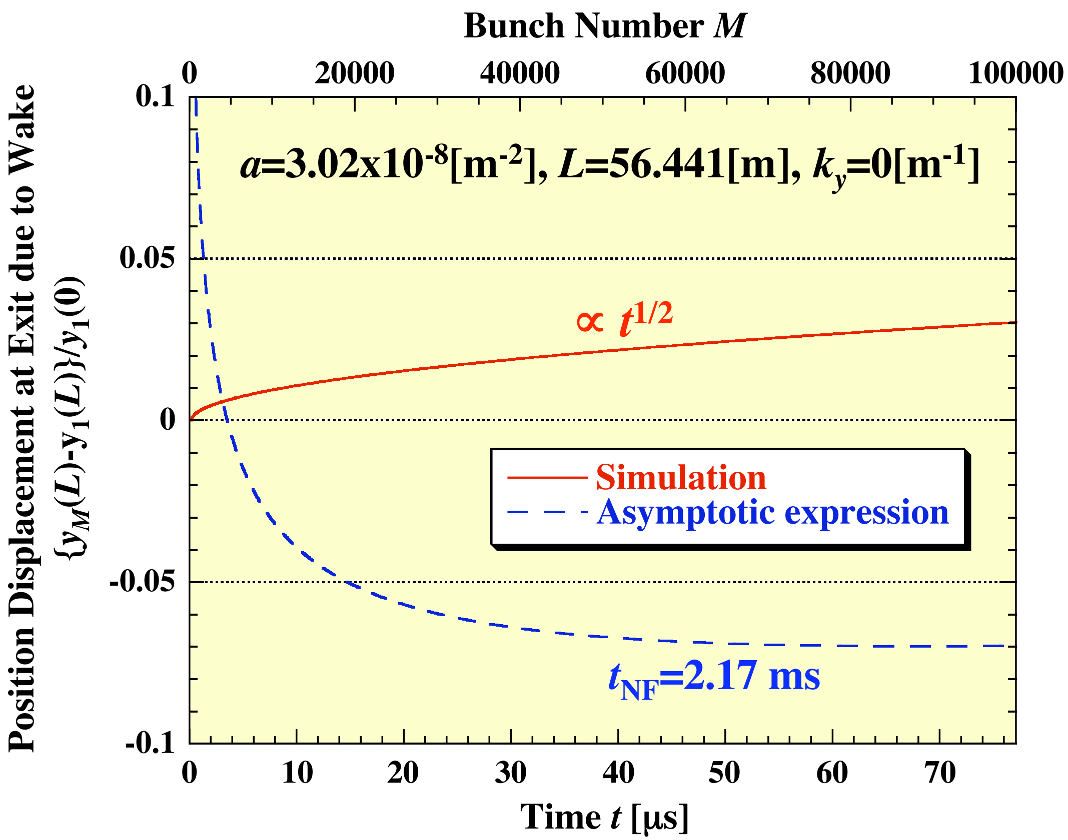
$$\xi_M{}''(s) \approx \sum_{N=1}^{M-1} \frac{a}{\sqrt{M-N}} \frac{y_N(s)}{y_1(0)} \approx 2a\sqrt{M} \frac{y_1(s)}{y_1(0)} \quad (M >> 1)$$

$$\xi_M(s) \equiv \frac{y_M(s) - y_1(s)}{y_1(0)} \propto a\sqrt{t} \quad \left(y_M{}'(0) = y_1{}'(0), y_M(0) = y_1(0), t = M\tau_B \right)$$

Position displacement due to RW wake is proportional to $t^{1/2}$ and a.

Simulation in ERL (3)

Q-magnets off (without insertion device)



- Position displacement due to wake increases in proportion to $t^{1/2}$ and up to 3 % of injection error at t = 77 μs.
- Asymptotic expression is invalid in the parameter range.

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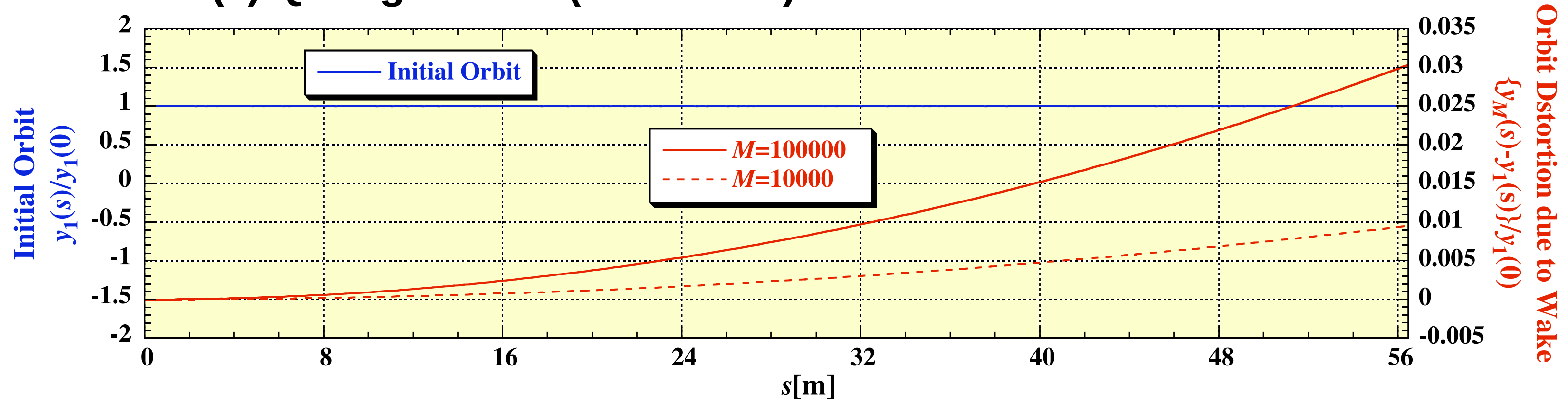
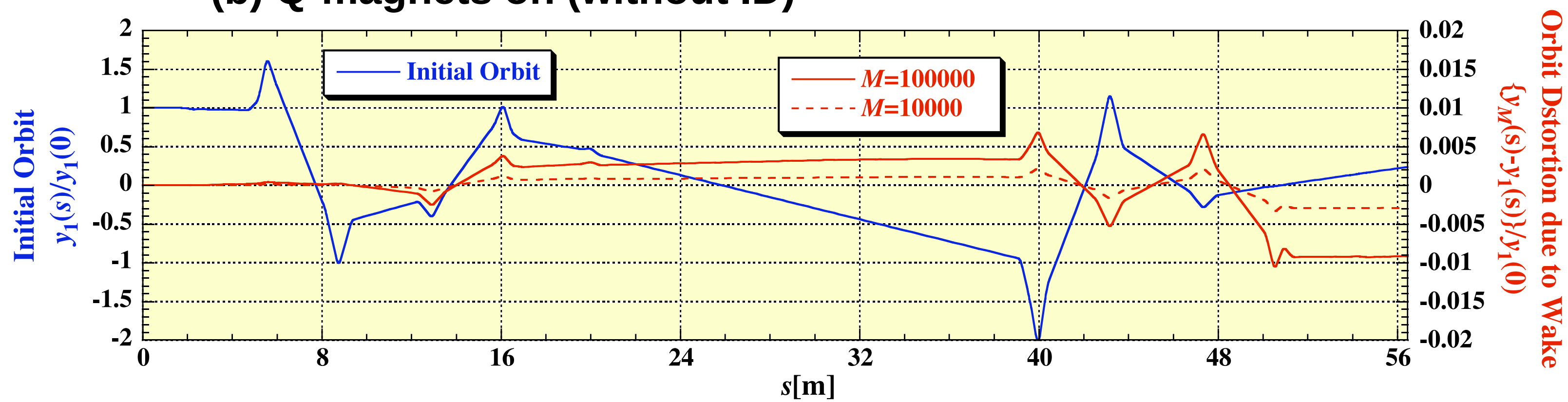
Simulation in ERL (4)

(a) Q-magnets off (without ID)



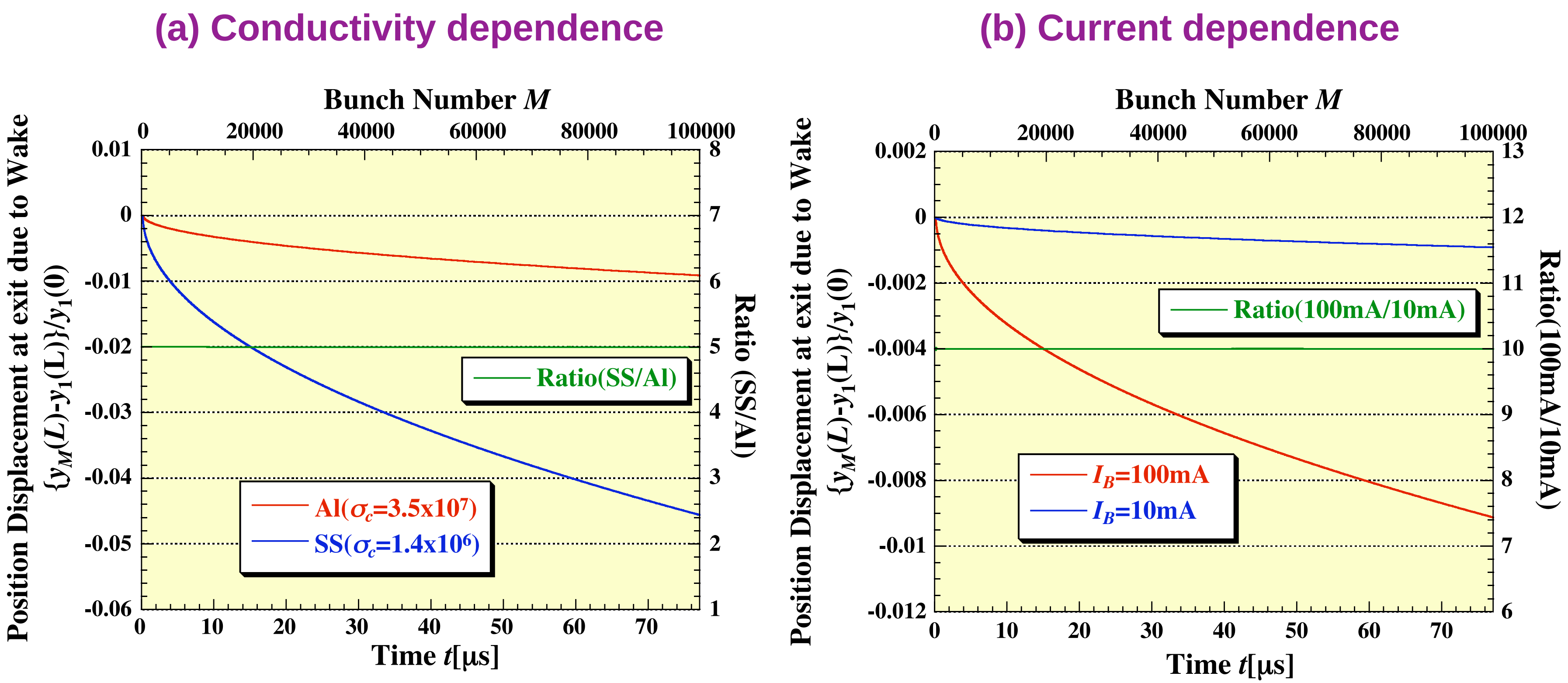
(b) Q-magnets on (without ID)



- Q-magnet focusing suppresses orbit distortion due to wake within 1% of injection error at 77 μs.
- Orbit correction is expected to be effective for suppressing the RWBBU.

Simulation in ERL (5)

Q-magnets on (without ID)



Position displacement is simply proportional to wake strength a ($\propto \sigma_c^{-1/2} I_B b^{-3} E^{-1}$) .

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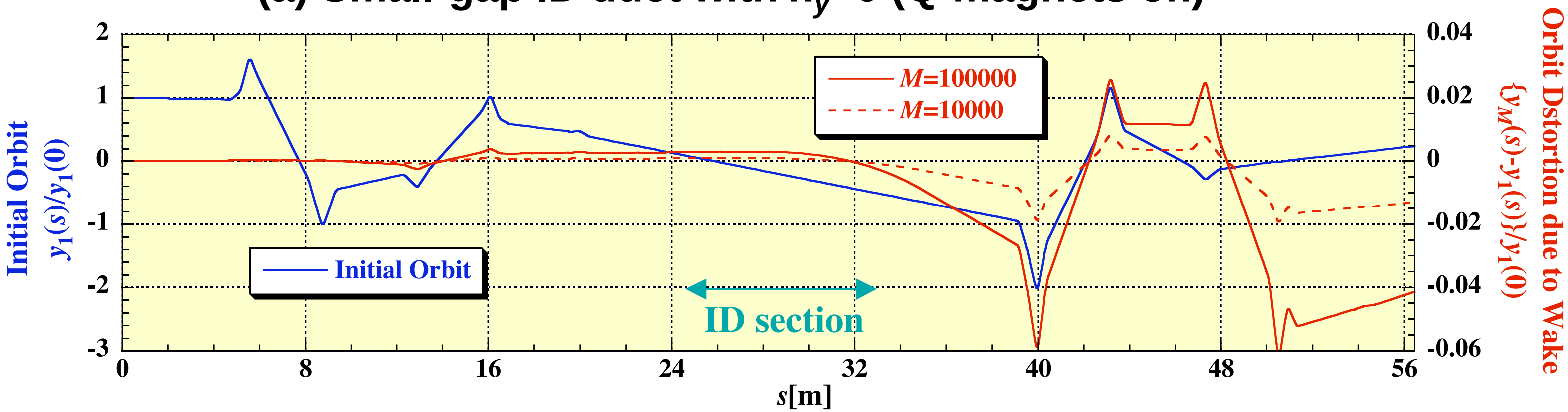
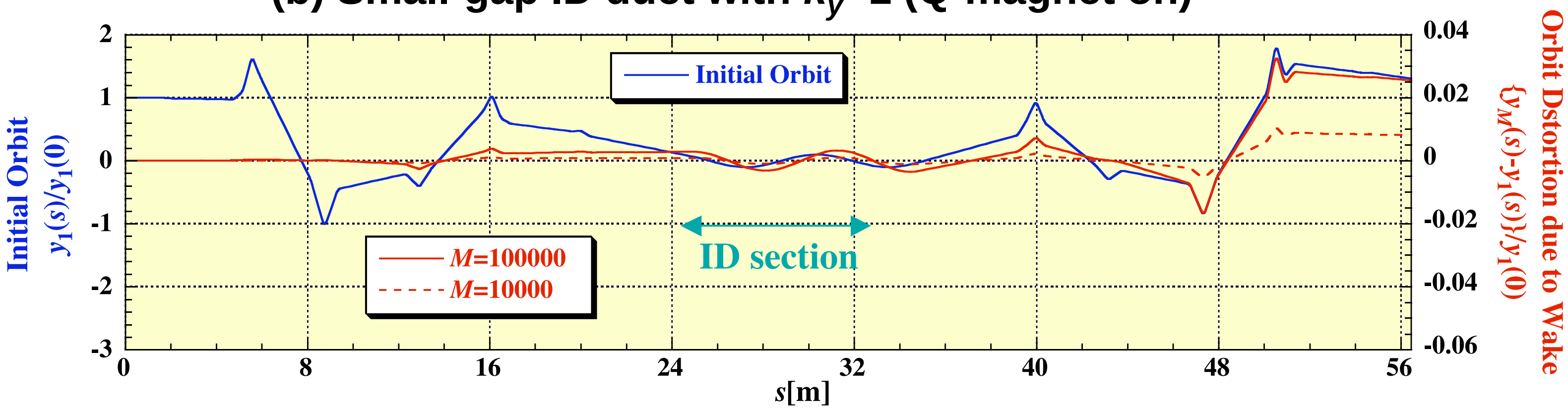
Simulation in ERL (6)

(a) Small-gap ID duct with k_y=0 (Q-magnets on)



(b) Small-gap ID duct with k_y=1 (Q-magnet on)



- A small gap ID duct significantly increases orbit distortion in the downstream area. Copper-coating on the inner surface can reduce the resistive-wall wake.
- ID focusing is effective for suppressing the orbit distortion but changeable.

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Summary and Conclusion

- We have developed a simulation program of RWBBU because the asymptotic expressions are valid only in the limited parameter ranges and initial conditions. The simulation program is useful for studying the transverse RWBBU more minutely and generally.
- The simulation program was applied to the test ERL in Japan. The maximum position displacements with all the Q-magnets off and on are about 3% and 1% of the injection error at 77 μs. The Q-magnet focusing contributes to slowing the RWBBU and orbit correction is also expected to be effective.
- A small-gap ID duct significantly increases orbit distortion. The ID focusing suppresses the RWBBU, but it is changeable.
- RWBBU should be studied experimentally and quantitatively with the test ERL in comparison with simulation results. Schemes of overcoming the RWBBU must be surveyed and studied.

Thank you for your attention and patience!